Arguments

Monty Python's "The Argument Clinic"

Definition: *Argument*

A connected series of statements to establish a definite proposition

Featuring:

Michael Palin	as	"Man"
Rita Davies	as	"Receptionist
Graham Chapman	as	"Mr. Barnard"
John Cleese	as	"Mr. Vibrating"
Eric Idle	as	"Complainer"
Terry Jones	as	"Spreaders"

https://www.youtube.com/watch?v=xpAvcGcEc0k

Inductive and Deductive Reasoning

Definition: *Inductive Argument*

An argument that moves form specific observations to general conclusions

Definition: *Deductive Argument*

An argument that uses accepted general principles to explain a specific situation

Inductive and Deductive Reasoning

- Example:
  - “Students who do well on the midterm do will in the class”

    $\Rightarrow$ **Inductive**

  - “Well-done hamburgers are safer to eat than medium-rare hamburgers”

    $\Rightarrow$ **Deductive (heat kills bacteria)**

Inductive and Deductive Reasoning

- What type of argument is this?
  - 3 is a prime number, 5 is a prime number, and 7 is a prime number. Therefore all odd integers above 1 are prime numbers

$\Rightarrow$ **Inductive**

(Specific examples lead to a general conclusion - that happens to be incorrect!)

Structure of a Deductive Argument

$$(p_1 \land p_2 \land \ldots \land p_n) \to q$$

- (Recall the form: Hypothesis → Conclusion)
- Approach: Logical principles are applied to the given hypothesis to see if the conclusion follows from them
- Common notations:
  - $p_1 \land p_2 \land \ldots \land p_n \,/\, \therefore q$ and

$$\begin{array}{l} \boxed{\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}} \ \textbf{Premises} \\ \hline \therefore \ q \end{array}$$

We will use the second one

Valid and Sound Arguments

Definition: *Valid Argument*

Any deductive argument of the form $(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q$ is valid if the conclusion must follow from the hypotheses

Definition: *Sound Argument*

A valid argument that also has true premises

Valid and Sound Arguments

- Example:

All men are mortal
Socrates is mortal

∴ All men are Socrates

Not a valid argument!

- Example:

Socrates is a man
All men are mortal

∴ Socrates is mortal

Valid! (Why?)

How can we show that arguments are valid?

- Truth tables
  - What if our argument has 10 propositions in it?
- Rules of Inference
  - Building blocks to construct (or validate) complicated arguments
  - Always valid no matter the premises and conclusion (but only sound when premises are true!)
  - Form:

$$
\begin{array}{l}
p_1 \\
p_2 \\
\vdots \\
p_n \\
\hline
\therefore\ q
\end{array}
$$

Corresponding propositional logic statement, $(p_1 \land p_2 \land \ldots \land p_n) \to q$, is always a tautology!

Rules of Inference

1. Modus Ponens (Method of affirming):

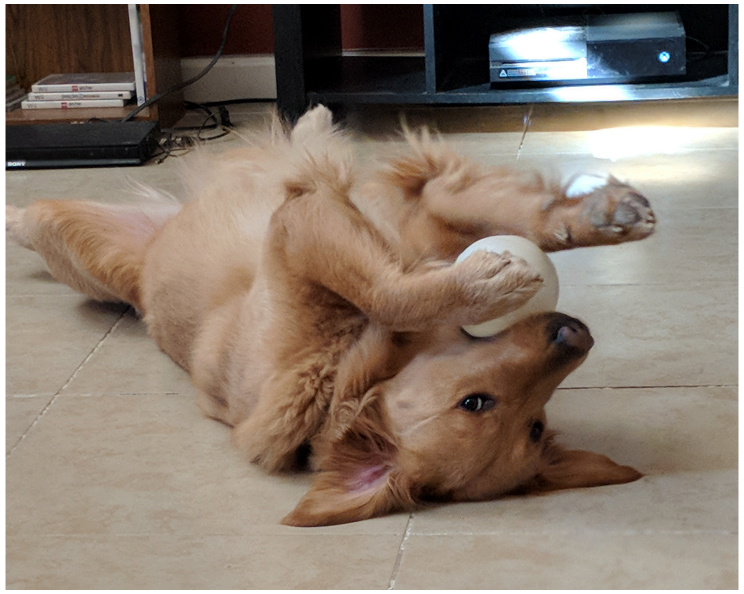
$$\begin{array}{l} \quad p \\ \quad p \to q \\ \hline \therefore q \end{array}$$

English: "If p, and p implies q, then q"

Propositional Logic: $(p \land (p \to q)) \to q$

Example: Modus Ponens

- Example: “Rodger squeaks his toy when he wants to play. He is squeaking his toy.” Does Rodger want to play?
  - s :Rodger squeaks his toy,
  - p : Rodger wants to play



$$\begin{array}{lll} (1) & s & (\text{Given}) \\ (2) & s \rightarrow p & (\text{Given}) \\ \hline \end{array}$$

Vertical structure leaves room for justification

Rules of Inference

2. Modus Tollens (Method of Denying):

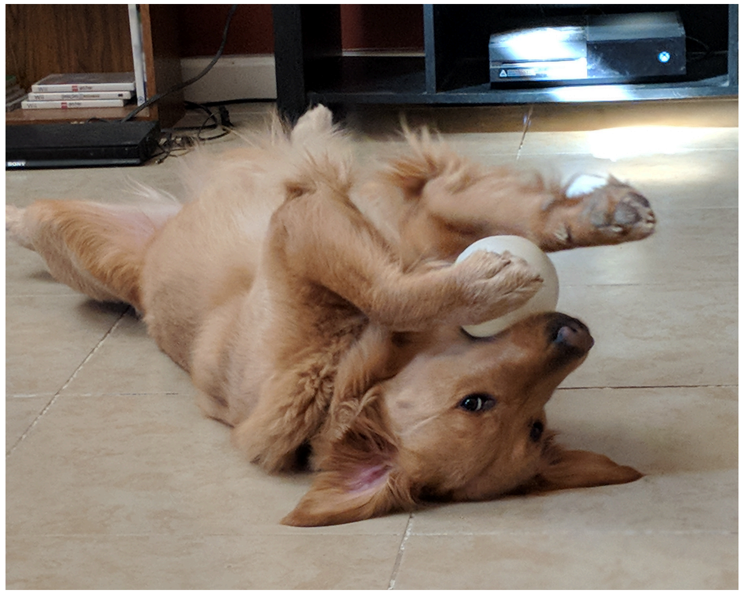
$$\begin{array}{l} \quad \neg q \\ \quad p \rightarrow q \\ \hline \therefore \neg p \end{array}$$

English: "If it is not the case that q, and p implies q, then it must not be the case that p"

Propositional Logic: $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

Example: Modus Tollens

- Example: “Rodger squeaks his toy when he wants to play. He does not want to play.” Is Rodger squeaking his toy?
  - s :Rodger squeaks his toy,
  - p : Rodger wants to play



$$
\begin{array}{lll}
(1) & \neg p & \text{(Given)} \\
(2) & s \to p & \text{(Given)} \\
\hline
\end{array}
$$

Rules of Inference

3. Hypothetical Syllogism (Transitivity of Implication):

$$\begin{array}{r} p \longrightarrow q \\ q \longrightarrow r \\ \hline \therefore p \longrightarrow r \end{array}$$

English: "If p implies q and q implies r, then p must imply r.

Propositional Logic:
$((p \to q) \land (q \to r)) \to (p \to r)$

Example: Hypothetical Syllogism

- Example: “If it’s spring, then there is pollen in the air. When there is pollen in the air, I sneeze.” If it’s spring, do I sneeze?
  - s :It is spring
  - p : There is pollen the air
  - z : I sneeze

$$
\begin{array}{lll}
(1) & s \rightarrow p & \text{(Given)} \\
(2) & p \rightarrow z & \text{(Given)} \\
\hline
\end{array}
$$

Rules of Inference

4. Disjunctive Syllogism (One or the Other):

$$\begin{array}{l} \quad p \lor q \\ \quad \neg p \\ \hline \therefore\ q \end{array}$$

English: “If p or q and it is not the case that p, then q.

Propositional Logic: $((p \lor q) \land \neg p) \to q$

5. Addition:

$$\begin{array}{l} \quad p \\ \hline \therefore\ p \lor q \end{array}$$

English: “If p, then p or q.

Propositional Logic: $p \to (p \lor q)$

Rules of Inference

6. Simplification:

$$\frac{p \wedge q}{\therefore\ p}$$

English: “If p and q, then p.

Propositional Logic: $(p \wedge q) \rightarrow p$

7. Conjunction:

$$\begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array}$$

English: “If p, and also q, then p and q.

Propositional Logic: $((p) \wedge (q)) \rightarrow (p \wedge q)$

Rules of Inference

8. Resolution:

$$\begin{array}{r} p \vee q \\ \neg p \vee r \\ \hline \therefore\ q \vee r \end{array}$$

English: "If p or q, and it is not the case that p or (it is the case that) r, then q or r"

Propositional Logic: $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$

- Why is this true?
  - If p is true, then $\neg p$ is false. Thus, by the second premise, r must be true.
  - If p is false, then $\neg p$ is true. Thus, by the first premise, p must be true
  - Thus, either p or r must be true.

Example: Resolution

- Example: "I walk my dog or it is raining. It is not raining or the wash is full of water ".
  - w : I walk my dog
  - r : It is raining
  - f : The wash is full

$$\begin{array}{lll} (1) & w \vee r & \text{(Given)} \\ (2) & \neg r \vee f & \text{(Given)} \\ \hline \end{array}$$

Summary: Rules of Inference

Learn These!

Name	Rule of Inference
Modus Ponens	$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$
Modus Tollens	$\begin{array}{l} \neg q \\ p \to q \\ \hline \therefore \neg p \end{array}$
Hypothetical Syllogism	$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$
Disjunctive Syllogism	$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$
Addition	$\begin{array}{l} p \\ \hline \therefore p \lor q \end{array}$
Simplification	$\begin{array}{l} p \land q \\ \hline \therefore p \end{array}$
Conjunction	$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$
Resolution	$\begin{array}{l} p \lor q \\ \neg p \lor r \\ \hline \therefore q \lor r \end{array}$

Using Rules of Inference

- Example 1: Is the argument below valid?
  - If 191 is divisible by 7, then 191^2 is divisible by 49. 191 is divisible by 7, so 191^2 must be divisible by 49.

Yes! The supporting rule is Modus Ponens:

a : 191 is divisible by 7

b : 191^2 is divisible by 49

$$
\begin{array}{lll}
(1) & a \to b & \text{(Given)} \\
(2) & a & \text{(Given)} \\
\hline
\end{array}
$$

Using Rules of Inference

- Example 2: If my advisor sends me an email at 10pm, I have to work late. If he doesn't, then I will get plenty of sleep. If I get plenty of sleep, I'll be more productive tomorrow.
- Show that if I don't work late, I will be productive tomorrow.

Begin by identifying propositions:

p :**My advisor sends me an email at 10pm**

q : **I work late**

r : **I get plenty of sleep**

s :**I am productive tomorrow**

Using Rules of Inference

- Example 2: If my advisor sends me an email at 10pm, I have to work late. If he doesn't, then I will get plenty of sleep. If I get plenty of sleep, I'll be more productive tomorrow.

p q r s

- Show that if I don't work late, I will be productive tomorrow.

Next, identify the givens and the desired conclusion

Givens

$$p \to q$$
$$\neg p \to r$$
$$r \to s$$

Conclusion

$$\therefore \neg q \to s$$

But how do we get there?

Using Rules of Inference

Propositions:	**Givens and Conclusion:**
p : My advisor sends me an email at 10pm	$p \rightarrow q$
q : I work late	$\neg p \rightarrow r$
r : I don't get much sleep	$r \rightarrow s$
s : I am productive tomorrow	$\therefore \neg q \rightarrow s$

$$
\begin{array}{llll}
(1) & & p \rightarrow q & \text{(Given)} \\
(2) & & \neg q \rightarrow \neg p & \text{(Contrapositive of (1))} \\
(3) & & \neg p \rightarrow r & \text{(Given)} \\
(4) & & \neg q \rightarrow r & \text{(Hypothetical Syllogism using (2) and (3))} \\
(5) & & r \rightarrow s & \text{(Given)} \\
\hline
(6) & \therefore & \neg q \rightarrow s & \text{(Hypothetical Syllogism using (4) and (5))}
\end{array}
$$

Note: This is a Formal Proof!

Using Rules of Inference

- Example 3: We go hiking today or its over 90 degrees. It is under 90 degrees or we go to Andrew's to play games. We do not go hiking or we go to Andrew's to play games. We won't eat too many cookies only if we don't go to Andrew's to play games.

- Show that we eat too many cookies.

p : **We go hiking**

q : **It is over 90 degrees**

r : **We go to Andrew's house to play games**

s : **We eat too many cookies**

Using Rules of Inference

- Example 3: We go hiking today or its over 90 degrees. It is under 90 degrees or we go to Andrew's to play games. We do not go hiking or we go to Andrew's to play games. We won't eat too many cookies only if we don't go to Andrew's to play games.

- Show that we eat too many cookies.

Givens

$$p \vee q$$

$$\neg q \vee r$$

$$\neg p \vee r$$

$$\neg s \rightarrow \neg r$$

Conclusion

$$\therefore ?$$

Using Rules of Inference

Propositions:	**Givens and Conclusion:**
p : We go hiking	$p \vee q$
q : It is over 90 degrees	$\neg q \vee r$
r : We go to Andrew's house to play games	$\neg p \vee r$
s : We eat too many of cookies	$\neg s \rightarrow \neg r$
	$\therefore ?$

(1)		$p \vee q$	(Given)
(2)		$\neg q \vee r$	(Given)
(3)		$p \vee r$	(Resolution of (1) and (2))
(4)		$\neg p \vee r$	(Given)
(5)		$r \vee r$	(Resolution of (3) and (4))
(6)		r	(Idempotent law on (5))
(7)		$\neg s \rightarrow \neg r$	(Given)
(8)		$\neg\neg s$	(Modus Tollens of (6) and (7))
(9)	$\therefore$	s	(Double negative equivalence of (8))

Additional Rules of Inference for Predicates

1. Universal Instantiation

- If we know something is true for the whole population (or domain D), we can conclude that it is true for a **specific** member of the group

$$\frac{\forall x P(x), x \in D}{\therefore\ P(d) \text{ if } d \in D}$$

Example

- Consider predicates:
  - $M(x)$: x is a man, $x \in$ people
  - $S(x)$: x is a mortal, $x \in$ people
- And the hypotheses:
  - All men are mortal ($\forall x\, M(x) \rightarrow S(x)$, $x \in$ people)
  - Socrates is a man (M(Socrates))

Example

Propositions:	**Givens and Conclusion:**
$M(x)$: x is a man, $x \in$ people	$\forall x\, M(x) \to S(X)$, $x \in$ **people**
$S(x)$: x is a mortal, $x \in$ people	$M($**Socrates**$)$
	$\therefore$?

$$
\begin{array}{llll}
(1) & & \forall x M(X) \to S(x),\ x \in \text{people} & \text{(Given)} \\
(2) & & M(Socrates) \to S(Socrates) & \text{(Universal Instantiation)} \\
(3) & & M(Socrates) & \text{(Given)} \\
\hline
(4) & \therefore & S(Socrates) & \text{(Modus Ponens of (2) and (3))}
\end{array}
$$

Additional Rules of Inference for Predicates

2. Universal Generalization

- If we know something is true for the an arbitrary element in the D and we've made no assumptions about that element, we can conclude that it is true for all elements in the D

$$\frac{P(d) \text{ for } \textbf{arbitrary } d \in D}{\therefore \forall x P(x), x \in D}$$

Caveat: Must ensure d is arbitrary!

Example

- Prove $\forall x Q(x)$ from the hypotheses:

1. $\forall x\ (P(x) \rightarrow Q(x)),\ x \in D$ (Given)
2. $\forall x\ P(x),\ x \in D$ (Given)
3. $P(c) \rightarrow Q(c)$, arbitrary $c \in D$ (Universal Instantiation)
4. $P(c)$ (Universal Instantiation)
5. $Q(c)$ (Modus Ponens of (3) and (4))

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6. $\therefore \forall x\ Q(x)$ (Universal Generalization)

Additional Rules of Inference for Predicates

3. Existential Instantiation

- We know some element, say d, such that $P(d)$ is true.

$$\frac{\exists x\, P(x), x \in D}{\therefore P(d) \text{ for some } d \in D}$$

- Here, d is a new name we give that specific element

  - d is **not** arbitrary, otherwise we would make false claims

  - e.g. “There exists an animal that flies, thus dogs flies.

Example

- Consider the hypotheses $\exists x\ P(x)$ and $\forall x\ \neg P(x)$. Show that we can derive a contradiction (i.e. **false**) from these hypotheses

1.	$\exists x\ P(x)$	(Given)
2.	$\forall x\ \neg P(x)$	(Given)
3.	$P(d)$	(Existential Instantiation)
4.	$\neg P(d)$	(Universal Instantiation)
5.	$P(d) \wedge \neg P(d)$	(Conjunction of (3) and (4))
6.	$\therefore$ **False**	(Negation Law)

Additional Rules of Inference for Predicates

4. Existential Generalization

- If we know that $P(d)$ is true for a specific element of D, then we know that there exists at least one element of D where $P(x)$ is true.

$$\frac{P(d) \text{ for some } d \in D}{\therefore \exists x \, P(x), x \in D}$$

Example

- $A(x)$: x attends UA
- $S(x)$: x is smart
- Given $A(George)$ and $S(George)$, prove that $\exists x\ (A(x) \wedge S(x))$

1.	$A(George)$	(Given)
2.	$S(George)$	(Given)
3.	$A(George) \wedge S(George)$	(Conjunction of (3) and (4))
4.	$\therefore \exists x\ (A(x) \wedge S(x))$	(Existential Generalization)

Summary of Rules of Inferences for Predicates

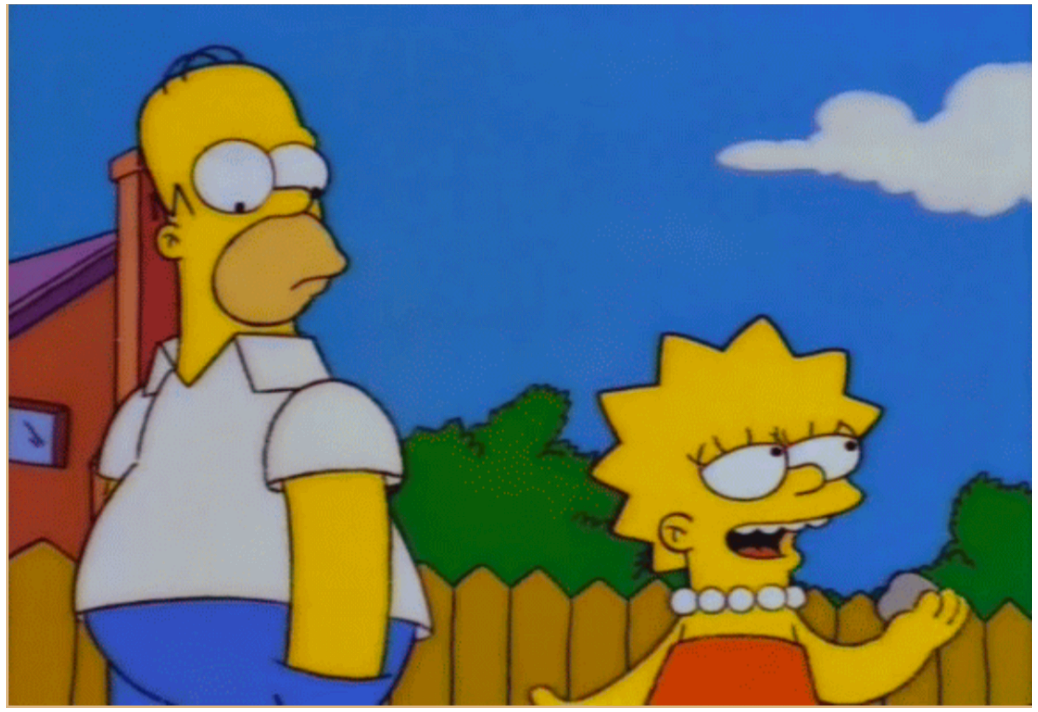
Name	Rule of Inference
Universal Instantiation	$\dfrac{\forall x P(x), x \in D}{\therefore P(d) \text{ if } d \in D}$
Universal Generalization	$\dfrac{P(d) \text{ for } \textbf{arbitrary } d \in D}{\therefore \forall x P(x), x \in D}$
Existential Instantiation	$\dfrac{\exists x\, P(x), x \in D}{\therefore P(d) \text{ for some } d \in D}$
Existential Generalization	$\dfrac{P(d) \text{ for some } d \in D}{\therefore \exists x\, P(x), x \in D}$

Example

- Prove that the following premises imply $\exists x\ (P(x) \land \neg B(x))$

1.	$\exists x\ (C(x) \land \neg B(x))$	(Given)
2.	$\forall x\ (C(x) \rightarrow P(x))$	(Given)
3.	$C(d) \land \neg B(d)$	(Existential Instantiation of (1))
4.	$C(d)$	(Simplification of (3))
5.	$C(d) \rightarrow P(d)$	(Universal Instantiation of (2))
6.	$P(d)$	(Modus Ponens of (4) and (5)
7.	$\neg B(d)$	(Simplification of (3))
8.	$P(d) \land \neg B(d)$	Conjunction of (6) and (7)
9.	$\exists x\ (P(x) \land \neg B(x))$	Existential Generalization of (8)

Specious Reasoning: The Bear Patrol



Homer:	Ah, not a bear in sight. The Bear Patrol must be working like a
Lisa:	That's **specious reasoning**, Dad. […] By your logic, I could claim that this rock keeps tigers away!
Homer:	Uh-huh, and how does it work?
Lisa:	It doesn't work. […] It's just a stupid rock […] But I don't see any tigers around here, do you?
Homer:	LIsa, I'd like to buy your rock.

From: The Simpsons, "Much Apu About Nothing"
(Season 7, Episode 151)

Specious Reasoning: The Bear Patrol

Definition: *Specious Reasoning*

An unsupported or improperly constructed argument. (That is, an unsound or invalid argument)

- Where is the error in Homer's logic?

b : There are bears in Springfield

w : The Bear Patrol is working

First Issue: Which of this is Homer's argument?

$$\begin{array}{lccc} (1) & & \neg b & (\text{Given}) \\ \hline (2) & \therefore & w & (???) \end{array} \qquad \begin{array}{lccc} (1) & & w & (\text{Given}) \\ \hline (2) & \therefore & \neg b & (???) \end{array}$$

The first seems most reasonable in this context

Specious Reasoning: The Bear Patrol

- Where is the error in Homer's logic?

Next, what is the missing piece of homers argument?

$$
\begin{array}{llll}
(1) & & \neg b & \text{(Given)} \\
(2) & & \neg b \rightarrow w & \\
\hline
(3) & \therefore & w & \text{(Modus Ponens from (1) and (2))}
\end{array}
$$

Whoops!
Unsupported implication

Valid
Argument

Specious Reasoning: The Bear Patrol

- Where is the error in Homer's logic?

Next, what is the missing piece of homers argument?

$$\begin{array}{llll} (1) & & \neg b & \text{(Given)} \\ (2) & & \neg b \to w & \\ \hline (3) & \therefore & w & \text{(Modus Ponens from (1) and (2))} \end{array}$$

Whoops!
Unsupported implication

Valid
(but unsound)
Argument

Ok, then how about…

$$\begin{array}{llll} (1) & & \neg b & \text{(Given)} \\ (2) & & w \to \neg b & \\ \hline (3) & \therefore & w & \text{(??? from (1) and (2))} \end{array}$$

Invalid
Argument

(the second form of Homer's argument fails similarly)

Fallacies

Definition: *Fallacy* (a form of specious reasoning)

A fallacy is an argument constructed with an improper inference.

- Three classic types:

1. Affirming the conclusion (or consequent)

Ex: If Juan is in Dallas, then he is in Texas. He is in Texas. Therefore, he is in Dallas.

$$\begin{array}{ll} p \to q & \text{(Given)} \\ q & \text{(Given)} \\ \hline \therefore p & \text{(???)} \end{array}$$

Fallacies

2. Denying the Hypothesis (or Antecedent)

Ex: If Ingrid is in Dallas, then she is in Texas. She is not in Dallas. Therefore, she is not in Texas.

$$\begin{array}{ll} p \to q & \text{(Given)} \\ \neg p & \text{(Given)} \\ \hline \therefore \neg q & \text{(???)} \end{array}$$

3. Begging the Question (or Circular Reasoning)

Ex: (1) I am not lying, so I must be telling the truth

(2) The law is the law

$$\begin{array}{ll} p & \text{(Given)} \\ \hline \therefore\ p & \text{(???)} \end{array}$$

Fallacies for Fun

1. Fallacy of Interrogation

Two classic examples:

- Have you stopped beating your spouse?
- Did you give your accomplice the stolen money?

2. No ‘True Scotsman’ Fallacy

“No American opposes tax cuts”

"Wendell is an American; he opposes them”

“No true American opuses tax cuts.