

CSc 245 Discrete Structures - Summer 2020
Homework #1 - Solutions

Questions

1. Email
2. Fractions: Simplify the following fractions

$$\begin{aligned} \text{(a)} \quad & \frac{-(\frac{x}{5}+2)}{4} * \frac{x}{8} \\ &= \frac{-x(\frac{x}{5}+2)}{32} \\ &= -\frac{x(\frac{x+10}{5})}{32} \\ &= -\frac{x(\frac{x+10}{5})}{32} \\ &= -\frac{x(x+10)}{160} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{2x}{5} - \frac{(x+9)}{7} \\ &= \frac{14x}{35} - \frac{5(x+9)}{35} \\ &= \frac{14x-5(x+9)}{35} \\ &= \frac{14x-5x-45}{35} \\ &= \frac{9x-45}{35} \end{aligned}$$

3. Rational Numbers: Determine if the following are rational numbers. If so, state the number as a ratio of two integers. If not, briefly explain why. (For guidance, refer to Section A.2, specifically Example 199, in the math review excerpt from Dr. McCann's book).

(a) 9.72727272727...

Yes. Let $x = 9.72727272727\dots$
 $10000x = 97272.72727272727\dots$
 $100x = 972.72727272727\dots$
 $10000x - 100x = 96300$
 $9900x = 96300$
 $x = \frac{96300}{9900} = \frac{107}{11}$

(b) 1.57079632679... No, the decimal never repeats so it cannot be rational.

4. Sets (pt 1) Write the resulting sets:

(a) $\mathbb{Z}^+ \cup \mathbb{Z}^* = \mathbb{Z}^*$

(b) $\mathbb{Z}^{odd} \cup \mathbb{Z}^{even} = \mathbb{Z}$

(c) $\mathbb{Z}^{odd} \cap \mathbb{Z}^{even} = \emptyset$

(d) $\mathbb{Z}^* - \mathbb{Z}^+ = \{0\}$

5. Sets (pt 2) Write true or false for each of the following:

(a) If $k \in \mathbb{Z}$, then $2k \in \mathbb{Z}^{even}$ - TRUE

- (b) If $2k \in \mathbb{Z}$, then $k \in \mathbb{Z}^{even}$ - FALSE
6. Sets (pt 3) Let $A = \{\alpha\}$ and $B = \{\beta, \gamma\}$ be sets from the universe $\mathcal{U} = \{\alpha, \beta, \gamma, \delta\}$. Write the following sets:
- $A - B = A = \{\alpha\}$
 - $\mathcal{U} - A = \{\beta, \gamma, \delta\}$
 - $(A \cup B) = \{\alpha, \beta, \gamma\}$
 - $\overline{A} = \{\beta, \gamma, \delta\}$
7. Associative, Commutative, and Distributive Properties.
- Expand $(z - y + 4)x = zx - zy + 4x$
 - Simplify $5(3x - 4) + 7(2y - x + 2)$
 $= 15x - 20 + 14y - 7x + 2$
 $= 8x - 18 + 14y$
 $= 2(4x - 9 + 7y)$
8. Properties of Inequalities (part 1): Determine if each inequality is True or False
- $-4 < -5$ - FALSE
 - $34 \geq 34$ - TRUE
 - $16 < 16$ - FALSE
9. Properties of Inequalities (part 2): Solve the following equations for x
- $x + 4 < 6x$
 $4 < 5x$
 $\frac{4}{5} < x$
 - $6 - 3x \geq 12$
 $-3x \geq 6$
 $x \leq -2$
10. Summation and Product Notations: Evaluate the following expressions.
- $\sum_{i=0}^5 (i^2 + 2)$
 $= (0 + 2) + (1 + 2) + (4 + 2) + (9 + 2) + (16 + 2) + (25 + 2)$
 $= 67$
 - $\prod_{i=2}^4 i + 1$
 $= 2 * 3 * 4 + 1$
 $= 25$
11. Integer Division (pt 1): Evaluate the following, giving a non-negative answer
- Evaluate $17 \% 7 = 3$.
 - Evaluate $-12 \% 13 = 1$.
 - Evaluate $4 \% 17 = 4$.
12. Integer Division (pt 2): Specify if the following statements are true or false.

- (a) $2 \equiv 18 \pmod{4}$ - TRUE
- (b) $-3 \equiv 3 \pmod{5}$ - FALSE
- (c) $-3 \equiv 3 \pmod{6}$ - TRUE
- (d) $m + 1 \equiv m - 1 \pmod{2}$ where $m \in \mathbb{Z}$ - TRUE

13. Integer Division (pt 3):

- (a) Evaluate $3|39$. - TRUE
- (b) Give 3 integers that are congruent to 23, modulo 5.
e.g. 3, 13, 18

14. Exponents and Logarithms (pt 1): Evaluate the following expressions, show your work.

- (a) $\log_2 32$
 $= \log_2 2^5$
 $= 5$
- (b) $\log_6 216 - \log_6 36$
 $= \log_6 \frac{216}{36}$
 $= \log_6 6$
 $= 1$
- (c) $\log_{11} \left(\frac{1}{11}\right)$
 $\log_{11} 11^{-1}$
 $= -1$
- (d) $\frac{\log_6 9}{\log_6 3}$
 $= \log_3 9$
 $= 2$

15. Exponents and Logarithms (pt 2): Simplify the expressions to use exactly one exponent.

- (a) $6^3 \cdot 7^3 = 42^3$
- (b) $3^6 \cdot 3^7 = 3^{13}$

16. Exponents and Logarithms (pt 3): Solve.

- (a) $\log_3 9^x = 5$
 $3^5 = 9^x$
 $(3^2)^{2.5} = 9^x$
 $9^{2.5} = 9^x$
 $x = 2.5$
- (b) $5^{\log_x 5} = 25$
 $\log_5 25 = \log_x 5$
 $2 = \log_x 5$
 $x^2 = 5$
 $x = \sqrt{5}$

17. Factoring Quadratics: Find the roots of the following equations by factoring

- (a) $6x^2 + 13x + 6 = 0$
 $(3x + 2)(2x + 3) = 0$
 $x = -\frac{2}{3}, -\frac{3}{2}$
- (b) $2x^2 - 3x - 2 = 0$
 $(2x + 1)(x - 2) = 0$
 $x = -\frac{1}{2}, 2$
- (c) $x^2 - x = 0$
 $x(x - 1) = 0$
 $x = 0, 1$

18. Number systems: Each value below is in either Binary, Octal, Decimal, or Hexadecimal. Convert each value to the 3 forms it is not given in.

- (a) 123_{10} Binary: 1111011_2 , Octal: 173_8
Binary:
 $1/2 = 0 \text{ R } 1$
 $3/2 = 1 \text{ R } 1$
 $7/2 = 3 \text{ R } 1$
 $15/2 = 7 \text{ R } 1$
 $30/2 = 15 \text{ R } 0$
 $61/2 = 30 \text{ R } 1$
 $123/2 = 61 \text{ R } 1$
 $= 1111011_2$
Octal: $1111011_2 = 001 \ 111 \ 011 = 173_8$
Hex: $1111011_2 = 0111 \ 1101 = 7B_{16}$
- (b) 10010101011_2
Octal: $10010101011_2 = 010 \ 010 \ 101 \ 011 = 2253_8$
Hex: $10010101011_2 = 0100 \ 1010 \ 1011_2 = 4AB_{16}$
Decimal: 10010101011_2
 $= 1 * 2^{10} + 0 * 2^9 + 0 * 2^8 + 1 * 2^7 + 0 * 2^6 + 1 * 2^5 + 0 * 2^4 + 1 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0$
 $= 2^{10} + 2^7 * 2^5 + 2^3 + 2^1 + 2^0$
 $= 1195_{10}$
- (c) 6723_8
Binary: $6723_8 = 110111010011_2$
Hexadecimal: 110111010011_2
 $= 1101 \ 1101 \ 0011_2 = DD3_{16}$
Decimal: 110111010011_2
 $= 1 * 2^{11} + 1 * 2^{10} + 0 * 2^9 + 1 * 2^8 + 1 * 2^7 + 1 * 2^6 + 0 * 2^5 + 1 * 2^4 + 0 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0$
 $= 2^{11} + 2^{10} + 2^8 + 2^7 + 2^6 + 2^4 + 2^1 + 2^0$
 $= 3539_{10}$
- (d) $A83C_{16}$
Binary: $A83C_{16} = 1010100000111100_2$
Octal: $1010100000111100_2 = 001 \ 010 \ 100 \ 000 \ 111 \ 100 = 124074_8$
Decimal: 1010100000111100_2

$$\begin{aligned} &= 1 * 2^{15} + 0 * 2^{14} + 1 * 2^{13} + 0 * 2^{12} + 1 * 2^{11} + 0 * 2^{10} + 0 * 2^9 + 0 * 2^8 + 0 * 2^7 + 0 * 2^6 + 1 * \\ &2^5 + 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 0 * 2^0 \\ &= 2^{15} + 2^{13} + 2^{11} + 2^5 + 2^4 + 2^3 + 2^2 \\ &= 43068 \end{aligned}$$