Quantification Examples

Converting From Quantified Predicates to Propositional Logic

Example 1:

Let $P(x)$: x is a prime number, $x \in \mathbb{Z}$

Express $\forall x\, P(x)$, $x \in \{3,5,7,9,11\}$ in propositional logic.

$$P(3) \land P(5) \land P(7) \land P(9) \land P(11)$$

What is its truth value?

Converting From Quantified Predicates to Propositional Logic

Example 2:

Let $P(x)$: x is a prime number, $x \in \mathbb{Z}$

Express $\neg \forall x\, P(x)$, $x \in \{3,5,7,9,11\}$ in propositional logic.

$$\neg(P(3) \wedge P(5) \wedge P(7) \wedge P(9) \wedge P(11))$$

$$\equiv \neg P(3) \vee \neg P(5) \vee \neg P(7) \vee \neg P(9) \vee \neg P(11)$$

What is its truth value?

Converting From Quantified Predicates to Propositional Logic

Example 3:

Let $P(x)$: x is a prime number, $x \in \mathbb{Z}$

Express $\exists x \neg P(x),\ x \in \{3,5,7,9,11\}$ in propositional logic.

$$\neg P(3) \vee \neg P(5) \vee \neg P(7) \vee \neg P(9) \vee \neg P(11)$$

Same proposition as $\neg \forall x\, P(x),\ x \in \{3,5,7,9,11\}$**!**
Generalized De Morgan's Laws

Converting From English to Quantified Predicates

Example 1: Express the following statement using Logic

"Some people in this class have seen Star Wars"

1. What are our predicates and their domains?

$S(x)$: x has seen Star Wars, $x \in$ People

2. What is our domain?

People in this class

2b. Does our domain create new predicates?

Yes! $C(x)$: x is in this class, $x \in$ People

3. What quantifier do we use?

$\exists x$

Converting From English to Quantified Predicates

Example 1: Express the following statement using Logic

"Some people in this class have seen Star Wars"

Putting it all together:

$S(x)$: x has seen Star Wars, $x \in$ People

$C(x)$: x is in this class, $x \in$ People

$$\exists x\,(C(x) \wedge S(x)),\ \ x \in \textbf{People}$$

Converting From English to Quantified Predicates

Example 2: Express the following statement using Logic

"All people in this class who have seen Star Wars think it's great"

1. What are our predicates and their domains?

$S(x)$: x has seen Star Wars, $x \in$ People

2. What is our domain?

People in this class

2b. Does our domain create new predicates?

Yes! $C(x)$: x is in this class, $x \in$ People

3. What quantifier do we use?

$\forall x$

Converting From English to Quantified Predicates

Example 2: Express the following statement using Logic

"All people in this class who have seen Star Wars think it's great"

Putting it all together:

$S(x)$: x has seen Star Wars, $x \in$ People

$G(x)$: x thinks Star Wars is great, $x \in$ People

$C(x)$: x is in this class, $x \in$ People

$$\forall x \; ((C(x) \wedge S(x)) \rightarrow G(x)), \;\; x \in \textbf{People}$$

Converting From Quantified Predicates to English

Example 1: Express the following statement in English

"$\forall x\,(C(x) \to (P(x) \land J(x))),\ x \in$ People"

Where $J(x)$: x knows Java

$P(x)$: x knows Python

$C(x)$: x is in this class

Everyone in this class knows Python and Java

Converting From Quantified Predicates to English

Example 2: Express the following statement in English

"$\forall x\,(C(x) \wedge P(x) \wedge J(x))$, $x \in$ People"

Where $J(x)$: x knows Java

$P(x)$: x knows Python

$C(x)$: x is in this class

All people are in this class and know Python and Java

Converting From Quantified Predicates to English

Example 3: Express the following statement in English

"$\exists x\,(C(x) \to (P(x) \wedge J(x)))$, $x \in$ People"

Where $J(x)$: x knows Java

$P(x)$: x knows Python

$C(x)$: x is in this class

For some person, if they are in this class, then they know Python and Java

Converting From Quantified Predicates to English

Example 4: Express the following statement in English

"$\exists x\,(C(x) \wedge P(x) \wedge J(x)),\ x \in$ People"

Where $J(x)$: x knows Java

$P(x)$: x knows Python

$C(x)$: x is in this class

Someone in this class knows Python and Java

Converting From English to Nested Quantifiers

Example 1: Express the following statement using Logic

If $x < y$, then $ax < ay$

1. What are our predicates and their domains?

$P(x, y) : x < y,\ x, y \in \mathbb{R}.\ Q(a, x, y) : ax < ay,\ x, y, a \in \mathbb{R}$

2. What is our domain?

$\mathbb{R}$

2b. Does our domain create new predicates?

No.

3. What quantifier(s) do we use?

$\forall x, \forall y, \forall a$

Converting From English to Quantified Predicates

Example 1: Express the following statement using Logic

If $x < y$, then $ax < ay$

Putting it all together:

$P(x, y) : x < y,\ \ x, y \in \mathbb{R}.$
$Q(a, x, y) : ax < ay,\ \ x, y, a \in \mathbb{R}$

$$\forall x \forall y \forall a \, (P(x, y) \to Q(a, x, y)),\ \ x, y, a \in \mathbb{R}$$

Note: The truth value of this statement is false. For this statement to be true, a needs to be positive!

Converting From English to Nested Quantifiers

Example 2: Express the following statement using Logic

"The difference of two positive integers

is not necessarily positive"

1. What are our predicates and their domains?

$P(x, y) : x - y > 0,\ \ x, y \in \mathbb{R}.\ Q(x) : x > 0,\ \ x \in \mathbb{R}$

2. What is our domain?

$\mathbb{Z}$

2b. Does our domain create new predicates?

No.

3. What quantifier(s) do we use?

$\exists x, \exists y$

Converting From English to Nested Quantifiers

Example 2: Express the following statement using Logic

"The difference of two positive integers
is not necessarily positive"

Putting it all together:

$P(x,y) : x - y > 0,\ x, y \in \mathbb{R}.$
$Q(x) : x > 0,\ x \in \mathbb{R}$

$$\exists x \exists y \left(Q(x) \wedge Q(y) \wedge \neg P(x,y)\right),\ x, y \in \mathbb{Z}$$

Converting From Nested Quantifiers to English

Example 1: Express the following statement in English

"$\exists x \forall y ((C(x) \wedge C(y)) \rightarrow F(x, y)),\ x, y \in$ People"

Where $C(x)$: x is in this class, $x \in$ People

$F(x, y)$: x and y are friends, $x, y \in$ People

Someone in this class is friends with everyone else in this class

Converting From Nested Quantifiers to English

Example 2: Express the following statement in English

"$\forall x \forall y \, ((C(x) \wedge C(y)) \rightarrow F(x, y)), \; x, y \in$ People"

Where $C(x)$: x is in this class, $x \in$ People

$F(x, y)$: x and y are friends, $x, y \in$ People

Everyone in this class is friends with everyone in this class

Converting From Nested Quantifiers to English

Example 3: Express the following statement in English

"$\exists x \, \exists y \, (C(x) \wedge C(y) \wedge F(x, y))$, $x, y \in$ People"

Where $C(x)$: x is in this class, $x \in$ People

$F(x, y)$: x and y are friends, $x, y \in$ People

Two people in this class are friends.

Note: the two people don't have to be different, they could be the same person

Converting From Nested Quantifiers to English

Example 4: Express the following statement in English

"$\forall x\,(C(x) \rightarrow \exists y\,(C(y) \wedge F(x,y))),\ x, y \in$ People"

Where $C(x)$: x is in this class, $x \in$ People

$F(x, y)$: x and y are friends, $x, y \in$ People

Everyone in this class is friends with someone in this class