**CSc 245 Discrete Structures - Summer 2020**

# Quiz #3

Due: June 30th, 2020 by 11:59 pm (MST)

Solutions

**A correct proof will:**

1. State proof type
2. Clearly state assumptions appropriate for stated proof type
3. Make justified steps toward conclusion. Justify steps with:
   (a) Math rules and definitions
   (b) Widely accepted, proven theorems (e.g. theorems from the book)
   (c) Previously proved conjectures (e.g. from lecture or prior assignments).
4. Clearly state the result/conclusion
5. Restate the conjecture at the end of the conclusion

1. Prove that if $3x + 5$ is even, then $x$ is odd.

   Proof (Contraposition): Assume $x$ is even.
   By definition, $\exists k \in \mathbb{Z}$ s.t. $x = 2k$.
   Substituting $2k$ for $x$ in $3x + 5$, we get $3(2k) + 5 = 6k + 5 = 2(3k + 4) + 1$
   $2(3k + 4) + 1$ can be rewritten as $2m + 1$ where $m = 3k + 4$, which is the form of an odd number.
   $m$ it is the product of two integers (3 and $k$) summed with another integer (4), which will be an integer.
   Thus, $3x + 5$ is odd and we have shown that if $x$ is even, then $3x + 5$ is odd.
   Therefore, by controposition, if $3x + 5$ is even, then $x$ is odd.

2. Prove that if $x$ is even and $y$ is odd, then $4|(y(x^2 + 2) - 6)$.

   Proof (Direct): Assume $x$ is even and $y$ is odd.
   By definition, $\exists k, j \in \mathbb{Z}$ s.t. $x = 2k$ and $y = 2j + 1$
   Substituting for $x$ and $y$ in $y(x^2 + 2) - 6$, we get:
   $(2j + 1)((2k)^2 + 2) - 6$
   $= (2j + 1)(4k^2 + 2) - 6$
   $= 8jk^2 + 4k^2 + 4j + 2 - 6$
   $= 8jk^2 + 4k^2 + 4j + -4$
   $= 4(2jk^2 + k^2 + j + -1)$
   Thus, since 4 can be factored out of the equation, then the equation is divisible by 4.
   Therefore, if $x$ is even and $y$ is odd, then $4|(y(x^2 + 2) - 6)$.