CSc 245 Discrete Structures - Summer 2020

Quiz #6

Due Tuesday July 28th at 11:59pm (MST)
Solutions

1. (4 points) Is the sequence $3, 2, 1, 0, 3, 2, 1, 0, 3, 2, 1, 0\ldots$ countable? Justify your answer.

   No it is not countable. The function $f(x) = 3 - x\%4$ maps the integers to the sequence but this function is not invertable because there are only 4 values in the range. Therefore, if we inverted it, each of those 4 integers would map to many numbers. Thus it is not countable.

2. (6 points) Prove using induction: $\sum_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2$

   Proof (weak induction):

   Base Case: n = 1. $\sum_{i=1}^{1} i2^i = 1 \cdot 2^1 = 2 = (1-1)2^{1+1} + 2$. So the Base Case holds.

   Inductive Case: If $\sum_{i=1}^{k} i2^i = (k-1)2^{k+1} + 2$, then $\sum_{i=1}^{k+1} i2^i = ((k+1)-1)2^{(k+1)+1} + 2 = k2^{k+2} + 2$.

   We know by definition of summations, that $\sum_{i=1}^{k+1} i2^i = \sum_{i=1}^{k} i2^i + (k+1)2^{k+1}$

   By the inductive hypothesis:

   $\sum_{i=1}^{k} i2^i + (k+1)2^{k+1} = (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$

   $= k2^{k+1} - 2^{k+1} + 2 + k2^{k+1} + 2^{k+1}$

   $= 2(k2^{k+1}) + 2$

   $= k2^{k+2} + 2$

   This is what we were trying to show.

   Therefore, $\sum_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2$.