# Course Background

(Why you’re here and what you learned to get here)

# &

# Math Review

# What is Discrete Math?

**Definition:** *Discrete Mathematics*

Discrete Mathematics is the study of collections of distinct objects

Contrast this with “the calculus”, which was developed by Newton and Leibniz to study objects in motion.  As a result:

- Calculus tends to focus on real values
- Discrete Mathematics tends to focus on integer values

# Sample Discrete Math Topics

Topics that fall under the umbrella of discrete math:

- Integral Functions and Relations
- Matrix Operations and Representations
- Sets
- Sequences and Summations
- Discrete Probability
- Counting (Permutations/Combinations, Recurrence Relations)

To understand those, you also need:

- First-Order logic
- Logical Arguments
- Proof Techniques
- … and a fair amount of pre-calculus mathematics

# How Discrete Math Relates to CS

Discrete Structures is an ACM/IEEE core curriculum topic

- See https://www.acm.org/binaries/content/assets/education/cs2013_web_final.pdf

DM topics underlie much of Computer Science, including:

- **Logic** -> Knowledge Representation, Reasoning, Natural Language Processing, Computer Architecture
- **Proof Techniques** -> Algorithm Design, Code Verification
- **Relations** -> Database Systems
- **Functions** -> Hashing, Programming Languages
- **Recurrence Relations** -> Recursive Algorithm Analysis
- **Probability** -> Algorithm Design, Simulation

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions

## Common Fractional Equivalencies:

(a) $\frac{x}{z} + \frac{y}{z} = \frac{x+y}{z}$ (b) $\frac{x}{z} - \frac{y}{z} = \frac{x-y}{z}$ (c) $\frac{x}{z}\frac{y}{z} = \frac{xy}{z^2}$ (d) $\frac{\frac{x}{z}}{\frac{y}{z}} = \frac{x}{y}$

(e) $\frac{x}{w} + \frac{y}{z} = \frac{xz+yw}{wz}$ (f) $\frac{x}{w} - \frac{y}{z} = \frac{xz-yw}{wz}$ (g) $\frac{x}{w}\frac{y}{z} = \frac{xy}{wz}$ (h) $\frac{\frac{x}{w}}{\frac{y}{z}} = \frac{xz}{wy}$

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions
- Rational Numbers

**Definition:** *Rational Number*

A value that can be expressed as the ratio of two integers

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions
- Rational Numbers
- Basics of Sets

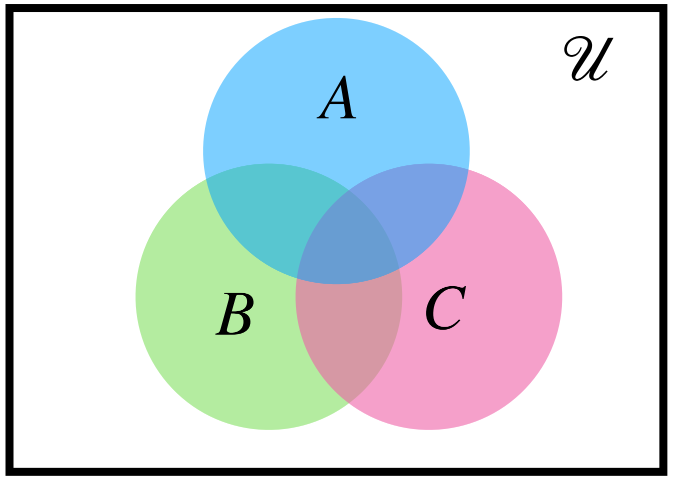
**Definition:** *Set*

An unordered collection of unique objets. $S = \{x_1, x_2, \ldots\}$

**Notation:** $s \in S$, $\emptyset$, $\mathscr{U}$, $\{x \,|\, f(x)$ **is true**$\}$

**Other Definitions:** Union, Intersection, Difference, Complement, Cardinality

**Venn Diagrams:**



# Notations for Sets of Values

| | | |
|---|---|---|
| $\mathbb{Z}$ | All integers | $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ |
| $\mathbb{Z}^+, \mathbb{N}^+$ | All positive integers | $\{1, 2, 3, \ldots\}$ |
| $\mathbb{Z}^*, \mathbb{N}_0$ | All non-negative integers | $\{0, 1, 2, 3, \ldots\}$ |
| $\mathbb{Z}^{even}$ | Even integers | $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$ |
| $\mathbb{Z}^{odd}$ | Odd integers | $\{\ldots, -3, -1, 1, 3, \ldots\}$ |
| $\mathbb{Q}$ | Rational numbers | $\frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0$ |
| $\overline{\mathbb{Q}}$ | Irrational Numbers | $\{i \mid i \notin \mathbb{Q}\}$ |
| $\mathbb{R}$ | The real values | $\{\mathbb{Q} \cup \overline{\mathbb{Q}}\}$ |

**Note: Avoid the term "natural numbers" and the symbol** $\mathbb{N}$

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions
- Rational Numbers
- Basics of Sets
- Associative, Commutative, Distributive, and Transitive Laws

**Definitions:** Associative, Commutative, Distributive, and Transitive

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions
- Rational Numbers
- Basics of Sets
- Associative, Commutative, Distributive, and Transitive Laws
- Properties of Inequalities

**Rules for adding/subtracting, multiplying/dividing**

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions
- Rational Numbers
- Basics of Sets
- Associative, Commutative, Distributive, and Transitive Laws
- Properties of Inequalities
- Summation and Product Notation

$$\sum_{i=0}^{k} s(i) \qquad \prod_{i=0}^{k} s(i)$$

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Fractions
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- Basics of Sets
- Associative, Commutative, Distributive, and Transitive Laws
- Properties of Inequalities
- Summation and Product Notation
- Integer Division (Modulo, Divides, Congruences)

# Integer Division

**Definition:** *Division*

(Our standard definition of division, denoted $a/b$ )

**Definition:** *Integer Division*

Integer division, denoted $a \backslash b$, returns the integer $m$ such that $a = m \cdot b + r$ where $r$ is the remainder

**Examples:**

- $10 \backslash 4$ $\quad 10 \backslash 4 = 2, \quad 10 = 2 * 4 + 2$
- $1 \backslash 5$ $\quad 1 \backslash 5 = 0, \quad 1 = 0 * 5 + 1$
- $13 \backslash 5$ $\quad 13 \backslash 5 = 2, \quad 13 = 2 * 5 + 3$

# Integer Division

**Definition:** *Modulo*

Denoted by % or **mod**, the modulus operator gives the remainder of an integer division. This is expressed as $a \% b = r$ , where $r$ is the remainder when $a$ is divided by $b$. In other words,

$a = m \cdot b + r, \quad (a, b, r, m) \in \mathbb{Z}$

**Examples:**

- $10 \% 4$ $\quad 10 \% 4 = 2$, $\ 10 = 2 * 4 + 2$
- $1$ **mod** $5$ $\quad 1$ **mod** $5 = 1$, $\ 1 = 0 * 5 + 1$
- $13 \% 5$ $\quad 13 \% 5 = 3$, $\ 13 = 2 * 5 + 3$

# Integer Division

**Definition:** *Congruency*

$a$ is congruent to $b$ modulo $m$, denoted $a \equiv b$ **mod** $m$, if $a \,\%\, m = b \,\%\, m$, or $(a - b) \,\%\, m = 0$.

In other words:

If $a \,\%\, m = r_1$ and $b \,\%\, m = r_2$, then $r_1 = r_2$

From here, we get the second form($(a - b) \,\%\, m = 0$):

Let $a = c \cdot m + r$ and $b = d \cdot m + r$ where $r = r_1 = r_2$

So $a - b = c \cdot m + r - (d \cdot m + r) = (c - d) \cdot m$ which is clearly divisible by $m$

# Integer Division

**Definition:** Congruency

$a$ is congruent to $b$ modulo $m$, denoted $a \equiv b$ **mod** $m$, if $a \,\%\, m = b \,\%\, m$, or $(a - b) \,\%\, m = 0$.

**Examples:**

- Is $10 \equiv 4$ **mod** $3$?

  **`True!`**

$$10 \,\%\, 3 = 1 \quad (10 = 3 * 3 + 1)$$
$$4 \,\%\, 3 = 1 \quad (4 = 3 * 1 + 1)$$
$$(10 - 4) \,\%\, 3 = 0$$

- Is $-3 \equiv 3$ **mod** $5$ **`False.`**

$$-3 \,\%\, 5 = 2 \quad (-3 = -1 * 5 + 2)$$
$$3 \,\%\, 5 = 3 \quad (3 = 5 * 0 + 3)$$
$$(-3 - 3) \,\%\, 5 \neq 0$$

# Integer Division

**Definition:** *Divides*

The "divides" operator, denoted $a \mid b$, returns **`True`** if $b \,\%\, a = 0$ and **`False`** otherwise.

**Examples:**

- $6 \mid 12$ $\qquad 12 \,\%\, 6 = 0$, so it is **`True`**
- $12 \mid 6$ $\qquad 6 \,\%\, 12 = 6$, so it is **`False`**
- $4 \mid 10$ $\qquad 10 \,\%\, 4 = 2$, so it is **`False`**

# Playposit Question:

Which of the following are **True**:

A. $5 \mid 10$

B. $10 \mid 5$

C. $5 \,\%\, 3 = 1$

D. $5 \backslash 3 = 1$

E. $13 \equiv 7 \text{ mod } 3$

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Even and Odd Integers

**Definition:** *Even*

An integer $n$ is even if there exists an integer $k$ such that $n = 2k$

**Definition:** *Odd*

An integer $n$ is odd if there exists an integer $k$ such that $n = 2k + 1$

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Even and Odd Integers
- Logarithms and Exponents

## Laws of Logarithms and Exponents

(a) $w^{x+y} = w^x w^y$

(b) $(w^x)^y = w^{xy}$

(c) $v^x w^x = (vw)^x$

(d) $\frac{w^x}{w^y} = w^{x-y}$

(e) $\frac{v^x}{w^x} = (\frac{v}{w})^x$

(f) $\log_b(x^y) = y \log_b x$

(g) $\log_b(xy) = \log_b x + \log_b y$

(h) $\log_b(\frac{x}{y}) = \log_b x - \log_b y$

(i) $b^{\log_b x} = x$

(j) $\log_a x = \frac{\log_b x}{\log_b a}$

(k) If $b^y = x$, then $\log_b x = y$

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Even and Odd Integers
- Logarithms and Exponents
- Working with Quadratic Equations

**Definitions:** Quadratic equations, Factoring Quadratic Equations, Quadratic Formula

# Topics You May Need to Review

Mathematical concepts, including, but not limited to:

- Even and Odd Integers
- Logarithms and Exponents
- Working with Quadratic Equations
- Positional Number Systems

**Decimal:** Base 10, Digits 0-9

**Binary:** Base 2, Digits 0,1

**Octal:** Base 8, Digits 0-7

**Hexadecimal:** Base 16, Digits 0-9, A-F

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- Properties of Inequalities
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- Integer Division (Modulo, Divides, Congruences)
- Even and Odd Integers
- Logarithms and Exponents
- Working with Quadratic Equations
- Positional Number Systems

Read the Math Review handout on the course website

# Homework 1

- Due **THIS** Friday (6/11) at 11:59pm MST
  - Intended to be refresher on these math topics
  - If you are not comfortable with these topics, read the math review excerpt from Dr. McCann’s book, found on the webpage.