

CSc 245 Discrete Structures - Summer 2021

Homework #1

(70 points)

Due: June 11th, 2021 by 11:59 p.m.

Solutions

Questions

- (3 points) To ensure we are able to communicate via email, send me an email at rjfaust+sum21csc245@email.arizona.edu with the subject line “[Last Name] - Hw1 Q1” and the following in the body:
 - Your preferred name
 - Your math background
 - Your CS background and preferred programming language
 - What time zone you will be in for the course
 - (optional) Anything else you would like me to know
- (4 points) Fractions: Simplify the following fractions
 - $\frac{-(\frac{x}{3}+6)}{2} * \frac{x}{3} = \frac{-x(x+18)}{18}$
 - $\frac{7x}{6} - \frac{(x+2)}{3} = \frac{(5x-4)}{6}$
- (2 points) Rational Numbers: Determine if the following are rational numbers. If so, state the number as a ratio of two integers. If not, briefly explain why. (For guidance, refer to Section A.2, specifically Example 199, in the math review excerpt from Dr. McCann’s book).
 - 12.5555... It is rational, $\frac{113}{9}$
 - 1.414213562... It is not rational. The decimal digits do never repeat.
- (4 points) Sets (part 1) Write the resulting sets:
 - $\mathbb{Z}^* \cup \mathbb{Z}^- = \mathbb{Z}$
 - $\mathbb{Z}^+ \cap \mathbb{Z}^- = \emptyset$
 - $\mathbb{Z} - \mathbb{Z}^{odd} = \mathbb{Z}^{even}$
 - $\mathbb{Z}^* \cap \mathbb{Z}^+ = \mathbb{Z}^+$
- (4 points) Sets (part 2) Write true or false for each of the following:
 - If $k \in \mathbb{Z}$, then $2k + 1 \in \mathbb{Z}^{odd}$ True.
 - If $2k + 1 \in \mathbb{Z}$, then $k \in \mathbb{Z}^{even}$ False.

6. (4 points) Sets (part 3) Let $A = \{\alpha, \beta\}$ and $B = \{\beta, \gamma\}$ be sets from the universe $\mathcal{U} = \{\alpha, \beta, \gamma, \delta\}$. Write the following sets:

- (a) $A \cap B = \{\beta\}$
- (b) $A - B = \{\alpha\}$
- (c) $\mathcal{U} - (A \cup B) = \{\delta\}$
- (d) $\overline{B} = \{\alpha, \delta\}$

7. (8 points) Associative, Commutative, and Distributive Properties.

(a) (2 points) Give a real world, non-mathematical, example of a relationship where the transitive property holds.

There are many solutions to this.

e.g. The bird is smaller than the cat. The cat is smaller than the dog. Thus, the bird is smaller than the dog.

(b) (3 points) For each of the following operators, indicate True or False for whether each operator is associative. If False, provide an example to demonstrate this.

i. $A \cup B$ True.

ii. $A \cap B$ True.

iii. $A - B$ False.

Let $A = \{\alpha, \beta\}, B = \{\beta, \gamma\}, C = \{\beta, \delta\}$

$(A - B) - C = \{\alpha\} - C = \{\alpha\}$

$A - (B - C) = A - \{\gamma\} = \{\alpha, \beta\}$

(c) (3 points) For each of the following operators, indicate True or False for whether each operator is commutative. If False, provide an example to demonstrate this.

i. $A \cup B$ True.

ii. $A \cap B$ True.

iii. $A - B$ False.

Let $A = \{\alpha, \beta\}, B = \{\beta, \gamma\}$

$(A - B) = \{\alpha\}$

$(B - A) = \{\gamma\}$

8. (3 points) Properties of Inequalities (part 1): Determine if each inequality is True or False

(a) $-46 < -5$ True.

(b) $21 \leq 21$ True.

(c) $7 > 7$ False.

9. (4 points) Properties of Inequalities (part 2): Solve the following inequalities for x

(a) $2x + 3 > 3x$

$3 > x$

(b) $7 - 4x \leq 15$

$x \geq -2$

10. (4 points) Summation and Product Notations: Evaluate the following expressions.

(a) $\prod_{i=1}^5 (i + 2) = 2520$.

(b) $\sum_{i=3}^7 i^2 + 1 = 136$

11. (6 points) Integer Division (part 1): Evaluate the following, giving a non-negative answer
- (a) Evaluate $12 \% 7 = 5$
 - (b) Evaluate $4 \% 12 = 4$
 - (c) Evaluate $-7 \% 8 = 1$
12. (4 points) Integer Division (part 2): Specify if the following statements are true or false.
- (a) $3 \equiv 18 \pmod{5}$ **True.**
 - (b) $-5 \equiv 5 \pmod{3}$ **False.**
 - (c) $m - 1 \equiv m + 1 \pmod{2}$ where $m \in \mathbb{Z}$ **True.**
 - (d) $12 \equiv 15 \pmod{3}$ **True.**
13. (4 points) Integer Division (part 3):
- (a) Evaluate $4|24$ **True.**
 - (b) Give 3 integers that are congruent to 25, modulo 6 (i.e. find 3 different integer values of a , such that $a \equiv 25 \pmod{6}$).
Any value x , where $x \% 6 = 1$
14. (6 points) Exponents and Logarithms: Evaluate the following expressions, show your work.
- (a) $\log_9(3^5 * 3^7)$
 $= \log_9(3^{12})$
 $= \log_9((3^2)^6)$
 $= \log_9(9^6)$
 $= 6$
 - (b) $\log_7 343 - \log_7 49$
 $= \log_7\left(\frac{343}{49}\right)$
 $= \log_7(7)$
 $= 1$
 - (c) $\frac{\log_7 4^8 - \log_7 8^3}{\log_7 2}$
 $= \frac{\log_7 \frac{4^8}{8^3}}{\log_7 2}$
 $= \frac{\log_7 \frac{(2^2)^8}{(2^3)^3}}{\log_7 2}$
 $= \frac{\log_7 \frac{2^{16}}{2^9}}{\log_7 2}$
 $= \frac{\log_7 2^7}{\log_7 2}$
 $= \log_2 2^7$
 $= 7$

15. (4 points) Factoring Quadratics: Factor the following equations and find the roots.

(a) $18x^2 - 8 = 0$

$(6x + 4)(3x - 2)$, roots: $-\frac{2}{3}, \frac{2}{3}$

(b) $2x^2 + 4x = 0$

$2x(x + 2)$, roots: $0, -2$

16. (6 points) Number systems: Convert the 145 to Binary, Octal and Hexidecimal. Show your work.

Binary

$145/2 = 72 \text{ R } 1$

$72/2 = 36 \text{ R } 0$

$36/2 = 18 \text{ R } 0$

$18/2 = 9 \text{ R } 0$

$9/2 = 4 \text{ R } 1$

$4/2 = 2 \text{ R } 0$

$2/2 = 1 \text{ R } 0$

$1/2 = 0 \text{ R } 1$

10010001

Octal

$145/8 = 18 \text{ R } 1$

$18/8 = 2 \text{ R } 2$

$2/8 = 0 \text{ R } 2$

221

Hexadecimal

$145/16 = 9 \text{ R } 1$

$9/16 = 0 \text{ R } 9$

91