CSc 245 - Introduction to Discrete Structures
Summer 2021

Midterm Instructions

1. The exam will open on Thursday (7/8) at 12am and must be completed before 11:59pm on Friday (7/9).
2. You will only have **90 minutes** from the time you open the exam to complete it. In order to have enough time to complete the exam, you must start it before 10:29pm (MST) on July 9th.
3. This exam is open book/course materials, closed internet. Looking for answers online or asking for/using answers written by other people is a violation of academic integrity.
4. Exams must be completed **individually**. You may not discuss questions with other students or on Piazza nor may you use other student's course materials on the exam. Once you complete the exam, please postpone any questions about or discussion of the exam with peers or on Piazza, until after the exam closes on Friday.
5. Clarifications may be asked on Piazza, but please make them private. We will decide if they should be made public.
6. Make your answers as precise, concise and to the point as possible, while still answering the questions asked.
7. When typing long answers, don't worry too much about formatting your answers nicely. Just make sure that your answer is readable.
8. Show your work, where appropriate, for potential partial credit. Vague, incomplete, and/or ambiguous answers will not receive full credit. Note, for short answer questions, you are not expected to show your work.
9. Some questions will have multiple parts in the same Gradescope question. Make sure to answer all parts.
10. I have provided alternative options for math notations that you may use when writing your answers. I have included these in all relevant questions, but will include them here as well (on the next page). You are not required to use this notation but it may be helpful to familiarize yourself with it before starting the exam. If you choose not to use this notation, make sure the notation you do use is clear.
11. Be sure to have the equivalence tables and rules of inference handy (I've included them at the end of this document).

Alternative Notation: The following is notation that you may use in your exam answers. Notation like this "$$\cup$$" will insert latex statements into your answer. You are not required to use these notation options as long as the notation you use is clear.

- Union: $$cup$$ or u
- Intersection: $$cap$$ or &
- Set Difference: -
- Set Complement: $$\overline{S}$$, !S, or `~S`
- Universal Set: $$\mathcal{U}$$ or U (make sure we can tell the difference between this and union).
- Conjunction: $$\land$$ or &
- Disjunction: $$\lor$$ or V
- Negation: $$\neg$$, or !
- Implication: $$\to$$ or ->
- Existential Quantifier: $$\exists$$ or E
- Universal Quantifier - $$\forall$$ or A
- Square root: $$\sqrt{x}$$ or sqrt(x)
- Exponents: $$x^2$$ or x^2
- $\mathbb{Z}$: $$\mathbb{Z}$$ or Z
- Fractions: $$\frac{a}{b}$$ or a/b
- $\in$ - $$\in$$ or "in"
- Matrices: for the matrix $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, you may write each row on one line, separating elements by a space:

```
1 2
3 4
```

CSc 245 — Introduction to Discrete Structures
(McCann)

Last Revised: May 2014

The Page O' Logical Equivalences ("POLE")

Table I: Some Equivalences using AND ($\wedge$) and OR ($\vee$):

(a)	$p \wedge p \equiv p$ $p \vee p \equiv p$	Idempotent Laws
(b)	$p \wedge \mathbf{F} \equiv \mathbf{F}$ $p \vee \mathbf{T} \equiv \mathbf{T}$	Domination Laws
(c)	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
(d)	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws
(e)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
(f)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws
(g)	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws

Table II: Some More Equivalences (adding Negation ($\neg$)):

(a)	$\neg(\neg p) \equiv p$	Double Negation
(b)	$p \wedge \neg p \equiv \mathbf{F}$ $p \vee \neg p \equiv \mathbf{T}$	Negation Laws
(c)	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws

Table III: Still More Equivalences (adding Implication ($\rightarrow$)):

(a)	$p \rightarrow q \equiv \neg p \vee q$	Law of Implication
(b)	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Law of the Contrapositive
(c)	$\mathbf{T} \rightarrow p \equiv p$	"Law of the True Antecedent"
(d)	$p \rightarrow \mathbf{F} \equiv \neg p$	"Law of the False Consequent"
(e)	$p \rightarrow p \equiv \mathbf{T}$	Self-implication (a.k.a. Reflexivity)
(f)	$p \rightarrow q \equiv (p \wedge \neg q) \rightarrow \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \rightarrow q \equiv p \vee q$	
(h)	$\neg(p \rightarrow q) \equiv p \wedge \neg q$	
(i)	$\neg(p \rightarrow \neg q) \equiv p \wedge q$	
(j)	$(p \rightarrow q) \vee (q \rightarrow p) \equiv \mathbf{T}$	Totality
(k)	$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation Law (a.k.a. Currying)
(l)	$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$	
(m)	$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$	
(n)	$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$	
(o)	$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$	
(p)	$p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$	Commutativity of Antecedents

Table IV: Yet More Equivalences (adding Exclusive OR ($\oplus$) and Biimplication ($\leftrightarrow$)):

(a)	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biimplication
(b)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
(c)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(d)	$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Definition of Exclusive Or
(e)	$p \oplus q \equiv \neg(p \leftrightarrow q)$	
(f)	$p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

Notes:

1. p, q, and r represent arbitrary logical expressions. They may represent equivalent expressions (e.g., if $p \equiv q$, then by Absorption $p \wedge (p \vee p) \equiv p$).
2. $\mathbf{T}$ and $\mathbf{F}$ represent the logical values True and False, respectively.
3. These tables show many of the common and/or useful logical equivalences; this is not an exhaustive collection!

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore\ q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore\ \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore\ p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore\ q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore\ p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore\ p$	$(p \land q) \to p$	Simplification
p q $\therefore\ p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore\ q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore\ P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore\ \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore\ P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore\ \exists x P(x)$	Existential generalization