CSc 245 Discrete Structures - Summer 2021

Homework #1 (70 points)

Due: June 11th, 2021 by 11:59 p.m.

Instructions:

1. **Homework assignments are to be completed individually, not in groups.**

2. If you need help, take advantage of Piazza and office hours.

3. Assignments are to be submitted in PDF form via Gradescope. They may be typed (which is preferable and strongly recommended) or handwritten with each page scanned or photographed and compiled into a single PDF.

4. If you choose to handwritten your assignments, please write neatly. Illegible assignments may not be graded.

5. Extra credit will be given for typed homework. To make this easier, a `Latex` template will be provided for each assignment.

6. Show your work (when appropriate) for partial credit!

Questions

1. (3 points) To ensure we are able to communicate via email, send me an email at rjfaust+sum21csc245@email.arizona.edu with the subject line "[Last Name] - Hw1 Q1" and the following in the body:

   (a) Your preferred name

   (b) Your math background

   (c) Your CS background and preferred programming language

   (d) What time zone you will be in for the course

   (e) (optional) Anything else you would like me to know

2. (4 points) Fractions: Simplify the following fractions

   (a) $\frac{-(\frac{x}{3}+6)}{2} * \frac{x}{3}$

   (b) $\frac{7x}{6} - \frac{(x+2)}{3}$

3. (2 points) Rational Numbers: Determine if the following are rational numbers. If so, state the number as a ratio of two integers. If not, briefly explain why. (For guidance, refer to Section A.2, specifically Example 199, in the math review excerpt from Dr. McCann's book).

   (a) $12.5555\ldots$

   (b) $1.414213562\ldots$

4. (4 points) Sets (part 1) Write the resulting sets:

   (a) $\mathbb{Z}^* \cup \mathbb{Z}^-$

   (b) $\mathbb{Z}^+ \cap \mathbb{Z}^-$

   (c) $\mathbb{Z} - \mathbb{Z}^{odd}$

   (d) $\mathbb{Z}^* \cap \mathbb{Z}^+$

5. (4 points) Sets (part 2) Write true or false for each of the following:

   (a) If $k \in \mathbb{Z}$, then $2k + 1 \in \mathbb{Z}^{odd}$

   (b) If $2k + 1 \in \mathbb{Z}$, then $k \in \mathbb{Z}^{even}$

6. (4 points) Sets (part 3) Let $A = \{\alpha, \beta\}$ and $B = \{\beta, \gamma\}$ be sets from the universe $\mathcal{U} = \{\alpha, \beta, \gamma, \delta\}$. Write the following sets:

   (a) $A \cap B$

   (b) $A - B$

   (c) $\mathcal{U} - (A \cup B)$

   (d) $\overline{B}$

7. (8 points) Associative, Commutative, and Distributive Properties.

   (a) (2 points) Give a real world, non-mathematical, example of a relationship where the transitive property holds.

   (b) (3 points) For each of the following operators, indicate True or False for whether each operator is associative. If False, provide an example to demonstrate this.

      i. $A \cup B$

      ii. $A \cap B$

      iii. $A - B$

   (c) (3 points) For each of the following operators, indicate True or False for whether each operator is commutative. If False, provide an example to demonstrate this.

      i. $A \cup B$

      ii. $A \cap B$

      iii. $A - B$

8. (3 points) Properties of Inequalities (part 1): Determine if each inequality is True or False

   (a) $-46 < -5$

   (b) $21 \leq 21$

   (c) $7 > 7$

9. (4 points) Properties of Inequalities (part 2): Solve the following inequalities for x

   (a) $2x + 3 > 3x$

   (b) $7 - 4x \leq 15$

10. (4 points) Summation and Product Notations: Evaluate the following expressions.

   (a) $\prod_{i=1}^{5}(i+2)$

   (b) $\sum_{i=3}^{7}i^2+1$

11. (6 points) Integer Division (part 1): Evaluate the following, giving a non-negative answer

   (a) Evaluate $12\,\%\,7$.

   (b) Evaluate $4\,\%\,12$.

   (c) Evaluate $-7\,\%\,8$.

12. (4 points) Integer Division (part 2): Specify if the following statements are true or false.

   (a) $3 \equiv 18 (\bmod\,5)$

   (b) $-5 \equiv 5 (\bmod\,3)$

   (c) $m - 1 \equiv m + 1 (\bmod\,2)$ where $m \in \mathbb{Z}$

   (d) $12 \equiv 15 (\bmod\,3)$

13. (4 points) Integer Division (part 3):

   (a) Evaluate $4|24$.

   (b) Give 3 integers that are congruent to 25, modulo 6 (i.e. find 3 <u>different</u> integer values of a, such that $a \equiv 25\,(\bmod\,6)$).

14. (6 points) Exponents and Logarithms: Evaluate the following expressions, show your work.

   (a) $\log_9(3^5 * 3^7)$

   (b) $\log_7 343 - \log_7 49$

   (c) $\frac{\log_7 4^8 - \log_7 8^3}{\log_7 2}$

15. (4 points) Factoring Quadratics: Factor the following equations and find the roots.

   (a) $18x^2 - 8 = 0$

   (b) $2x^2 + 4x = 0$

16. (6 points) Number systems: Convert the 145 to Binary, Octal and Hexidecimal. Show your work.