CSc 245 Discrete Structures - Summer 2021

Homework #2

(55 points)

Due: June 18th, 2021 by 11:59 p.m (MST).

Solutions

Instructions:

1. **Homework assignments are to be completed individually, not in groups.**
2. If you need help, take advantage of Piazza and office hours.
3. Assignments are to be submitted in PDF form via Gradescope. They may be typed (which is preferable and strongly recommended) or handwritten with each page scanned or photographed and compiled into a single PDF.
4. If you choose to handwrite your assignments, please write neatly. Illegible assignments may not be graded.
5. Extra credit will be given for typed homework. To make this easier, a `Latex` template will be provided for each assignment.
6. Show your work (when appropriate) for partial credit!

1. (3 points) Which of the following are propositions? If they are propositions, give their truth value.
   (a) California is on the Atlantic coast.
   It is a proposition. False
   (b) Phoenix is the capital of Arizona.
   It is a proposition. True
   (c) If $1 + 1 = 3$, then $1 + 1 = 4$.
   It is a proposition. True
2. (2 points) For each proposition, state whether the disjunction is inclusive or exclusive.
   (a) During spring break, you can go to either Canada or Mexico for vacation.
   Excusive
   (b) You must have experience with Python or Java.
   Inclusive
3. (6 points) Give the negations for each of the following propositions.
   (a) Rosa is taller than Amy.
   Rosa is shorter than or the same height as Amy
   (b) Ilana has a degree in CS and a degree in psychology.
   Ilana doesn't have a CS degree or she doesn't have a psychology degree.
   (c) Lincoln wants to be a dentist or an orthodontist.
   Lincoln does not want to be a dentist and he does not want to be an orthodontist.
   Alternative answer if XOR was used: Lincoln wants to be a dentist or he doesn't want to be an orthodontist, and, Lincoln doesn't want to be a dentist or wants to be a orthodontist.

4. (4 points) For each of the following propositions, express them in the form "if p then q".

   (a) Jason only swims whenever it is not raining.
   If Jimmy swims, then it is not raining.

   (b) I charge my laptop whenever it gets below 30% .
   If my laptop gets below 30%, then I charge it.

5. (12 points) Let c : I will go camping, f: I will go fishing, and w : the weather is nice.

   (a) Translate the following to English.

      i. $w \to (c \oplus f)$
      If the weather is nice, then either I will go camping or I will go fishing.

      ii. $c \wedge (w \to f)$
      I will go camping and also if the weather is nice, I will go fishing

      iii. $(c \wedge w) \to \neg f$
      If I go camping and the weather is nice, then I will not go fishing

   (b) Translate the following to logic.

      i. I will go fishing provided that I don't go camping or that the weather is nice.
      $(\neg c \vee w) \to f$

      ii. I won't go camping unless the weather is nice and I won't go fishing unless I go camping.
      $(\neg w \to \neg c) \wedge (\neg c \to \neg f)$

      iii. The weather is nice so I am going fishing and camping. [**Update: Instead, treat this question as "When the weather is nice, I go fishing and camping"**]
      $w \to (f \wedge c)$

6. (4 points) Give the truth table for $(p \oplus q) \wedge r \to (\neg p \wedge r) \vee \neg q$. Be sure each column only adds 1 new operator (as discussed in lecture).

p	q	r	$\neg p$	$\neg q$	$(p \oplus q)$	$\neg p \wedge r$	$(p \oplus q) \wedge r$	$(\neg p \wedge r) \vee \neg q$	$(p \oplus q) \wedge r \to (\neg p \wedge r) \vee \neg q$
T	T	T	F	F	F	F	F	F	T
T	T	F	F	F	F	F	F	F	T
T	F	T	F	T	T	F	T	T	T
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	T	T	T	T	T
F	T	F	T	F	T	F	F	F	T
F	F	T	T	T	F	T	F	T	T
F	F	F	T	T	F	F	F	T	T

7. (6 points) For each of the following, determine if it's a tautology, contradiction, or contingency. Show your work.

   (a) $\neg(\neg(p \vee \neg q) \to (q \to \neg p))$
   Contradiction

p	q	$\neg p$	$\neg q$	$(p \vee \neg q)$	$\neg(p \vee \neg q)$	$(q \to \neg p)$	$(\neg(p \vee \neg q) \to (q \to \neg p))$	$\neg(\neg(p \vee \neg q) \to (q \to \neg p))$
T	T	F	F	T	F	F	T	F
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	F	T	F
F	F	T	T	F	T	T	T	F

An example using equivalences:

$$
\begin{aligned}
& \neg(\neg(p \vee \neg q) \to (q \to \neg p)) & \\
\equiv\; & \neg((\neg p \wedge q) \to (q \to \neg p)) & \text{(DeMorgan's Law)} \\
\equiv\; & \neg((\neg p \wedge q) \to (\neg q \vee \neg p)) & \text{(Law of Implication)} \\
\equiv\; & \neg(\neg(\neg p \wedge q) \vee (\neg q \vee \neg p)) & \text{(Law of Implication)} \\
\equiv\; & \neg((p \vee \neg q) \vee (\neg q \vee \neg p)) & \text{(DeMorgan's Law)} \\
\equiv\; & \neg((p \vee \neg p) \vee (\neg q \vee \neg q)) & \text{(Associative/Commutative Laws)} \\
\equiv\; & \neg(T \vee (\neg q \vee \neg q)) & \text{(Negation Laws)} \\
\equiv\; & \neg(T) & \text{(Domination Laws)} \\
\equiv\; & F & \text{(Def. of Negation)}
\end{aligned}
$$

(b) $((p \vee q) \to (\neg p \wedge q)) \to \neg p$

Tautology

p	q	$\neg p$	$p \vee q$	$(\neg p \wedge q)$	$((p \vee q) \to (\neg p \wedge q))$	$((p \vee q) \to (\neg p \wedge q)) \to \neg p$
T	T	F	T	F	F	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	F	T	F	F	T	T

An example using equivalences:

$$
\begin{aligned}
& ((p \vee q) \to (\neg p \wedge q)) \to \neg p & \\
\equiv\; & (\neg(p \vee q) \vee (\neg p \wedge q)) \to \neg p & \text{(Law of Implication)} \\
\equiv\; & ((\neg p \wedge \neg q) \vee (\neg p \wedge q)) \to \neg p & \text{(DeMorgan's Law)} \\
\equiv\; & (\neg p \wedge (q \vee \neg q)) \to \neg p & \text{(Distributive Law)} \\
\equiv\; & (\neg p \wedge (T)) \to \neg p & \text{(Negation Law)} \\
\equiv\; & (\neg p) \to \neg p & \text{(Identity Law)} \\
\equiv\; & T & \text{(Self-implication)}
\end{aligned}
$$

(c) $(p \wedge q) \to (\neg p \vee \neg q)$

Contingency

p	q	$\neg p$	$\neg q$	$(p \wedge q)$	$(\neg p \vee \neg q)$	$(p \wedge q) \to (\neg p \vee \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

An example using equivalences:

$$
\begin{aligned}
& (p \wedge q) \to (\neg p \vee \neg q) & \\
\equiv\; & \neg(p \wedge q) \vee (\neg p \vee \neg q) & \text{(Law of Implication)} \\
\equiv\; & (\neg p \vee \neg q) \vee (\neg p \vee \neg q) & \text{(DeMorgan's Law)} \\
\equiv\; & (\neg p \vee \neg p) \vee (\neg q \vee \neg q) & \text{(Associative/Commutative Laws)} \\
\equiv\; & (\neg p) \vee (\neg q) & \text{(Idempotent Laws)}
\end{aligned}
$$

We know $\neg p \vee \neg q$ will have both T and F values, so it is a contigency.

8. (2 points) Evaluate the following expression: $(10110 \wedge 11010) \oplus 101010$

$$
\begin{array}{rr}
 & 10110 \\
\wedge & 11010 \\
\hline
 & 10010
\end{array}
\qquad
\begin{array}{rr}
 & 010010 \\
\oplus & 101010 \\
\hline
 & 111000
\end{array}
$$

9. (6 points) Give, in English, the inverse, converse, and contrapositive of the following proposition: "I have energy for my classes when I drink coffee in the morning"

   Inverse: If I don't drink coffee in the morning, then I won't have energy for my classes.

   Converse: If I have energy for my classes, then I drank coffee in the morning.

   Contrapositive: If I don't have energy for my classes, then I did not drink coffee in the morning.

10. (6 points) Use the equivalence tables to show that $p \vee q \rightarrow \neg p \wedge q$ is equivalent to $\neg p$. Clearly label the rule used at each step (if the rule is not formally named, use the table number and row letter).

    Example Answer:

$$
\begin{array}{lll}
 & (p \vee q) \rightarrow (\neg p \wedge q) & \\
\equiv & \neg(p \vee q) \vee (\neg p \wedge q) & \text{(Law of Implication)} \\
\equiv & (\neg p \wedge \neg q) \vee (\neg p \wedge q) & \text{(DeMorgan's Law)} \\
\equiv & (\neg p \wedge (q \vee \neg q)) & \text{(Distributive Law)} \\
\equiv & (\neg p \wedge (T)) & \text{(Negation Law)} \\
\equiv & (\neg p) & \text{(Identity Law)}
\end{array}
$$

11. (4 points) Given the following code snippet, express the statements on lines 3-7 in English and propositional logic with respect to the call to "check_city" on line 11. Be sure to create clearly defined propositional variables for each atomic proposition in the statement.

```
1   public class Relocation{
2       public check_city(City c, Person p){
3           if ((c.population < 500,000 && c.region == "NE") ||
4           (c.isOnCoast && c.region == "NE")){
5               move_to_city(c,p)
6           }
7       }
8       public static void main(String []args){
9           City c = City("Boston")
10          Person p = Person("Michelle")
11          check_city(c,p)
12      }
13  }
```

    Let p : the population of Boston is less than 500,000.

    r : Boston is in the northeast.

    c : Boston is on the coast.

    m : Michelle moves to Boston.

    $((p \wedge r) \vee (c \wedge r)) \rightarrow m$

    English: If Boston has a population less than 500,000 and it is in the Northeast, or Boston is on the coast and is in the Northeast, then Michelle will move to Boston.