CSc 245 Discrete Structures - Summer 2021

Homework #3

(60 points)

Due: June 25th, 2021 by 11:59 p.m (MST).

TODO: Your Name Here

Instructions:

1. **Homework assignments are to be completed individually, not in groups.**
2. If you need help, take advantage of Piazza and office hours.
3. Assignments are to be submitted in PDF form via Gradescope. They may be typed (which is preferable and strongly recommended) or handwritten with each page scanned or photographed and compiled into a single PDF.
4. If you choose to handwritten your assignments, please write neatly. Illegible assignments may not be graded.
5. Extra credit will be given for typed homework. To make this easier, a `Latex` template will be provided for each assignment.
6. Show your work (when appropriate) for partial credit!

1. (8 points) Given the following predicates, convert each logic statement into English.

   $R(x)$: x costs at least \$600 per month to rent, $x \in$Houses

   $S(x)$: x has at least two bedrooms, $x \in$Houses

   (a) $\forall x\, R(x) \rightarrow S(x)$, $x \in$ Houses

   All houses that cost at least \$600 per month to rent have at least 2 bedrooms.

   (b) $\forall x\, R(x) \wedge S(x)$, $x \in$ Houses

   All houses cost at least \$600 per month to rent and have at least two bedrooms.

   (c) $\exists x\, R(x) \rightarrow S(x)$, $x \in$ Houses

   There exists a house where, if it costs at least \$600 to rent, then it has at least two bedrooms.

   (d) $\exists x\, R(x) \wedge S(x)$, $x \in$ Houses

   There is a house that costs \$600 per month to rent has at least 2 bedrooms.

2. (15 points) Complete the specified conversions given the following predicates.

   $T(x)$: x lives in Tucson, $x \in$ People

   $C(x)$: x is in this course, $x \in$ People

   $D(x)$: x has a dog, $x \in$ People

   $W(x)$: x likes hot weather, $x \in$ People

   (a) (6 points) Convert the following English statements to propositional logic.

      i. Someone in this course lives in Tucson but doesn't like hot weather.

      $\exists x\ (C(x) \wedge T(x) \wedge \neg W(x)),\ \ x \in$ People

ii. Everyone who lives in Tucson likes hot weather, has a dog or is in this course.
$\forall x\ T(x) \rightarrow (W(x) \vee D(x) \vee C(x)),\ \ x \in \text{People}$

iii. No one in this course both lives in Tucson and has a dog.
$\forall x\ C(x) \rightarrow \neg(T(x) \wedge D(x)),\ \ x \in \text{People}$
Alternative Answer: $\forall x\ C(x) \rightarrow (\neg T(x) \vee \neg D(x)),\ \ x \in \text{People}$

(b) (9 points) Negate your logic statements in part (a) so that the negations are fully distributed, and then convert them back to conversational English (i.e. For proposition p, do not simply state, "it is not the case that p"). Provide both the negated logic and English statements in your answer.

i. $\neg(\exists x\ (C(x) \wedge T(x) \wedge \neg W(x))),\ x \in \text{People}$
$\equiv \forall x\ \neg(C(x) \wedge T(x) \wedge \neg W(x)),\ x \in \text{People}$
$\equiv \forall x\ (\neg C(x) \vee \neg T(x) \vee W(x)),\ x \in \text{People}$
English: For all people at last one of the following is true: they are not in this class, they don't live in Tucson or they like hot weather.

Alternative: $\forall x\ C(x) \rightarrow (\neg T(x) \vee W(x)),\ \ x \in \text{People}$
Everyone in this class does not live in Tucson or likes the hot weather.

ii. $\neg(\forall x\ T(x) \rightarrow (W(x) \vee D(x) \vee C(s))),\ x \in \text{People}$
$\equiv \exists x\ \neg(T(x) \rightarrow (W(x) \vee D(x) \vee C(s))),\ x \in \text{People}$
$\equiv \exists x\ (T(x) \wedge \neg(W(x) \vee D(x) \vee C(s))),\ x \in \text{People}$
$\equiv \exists x\ (T(x) \wedge \neg W(x) \wedge \neg D(x) \wedge \neg C(s)),\ x \in \text{People}$
English: Someone who lives in Tucson is not in this class, doesn't have a dog, and does not like the hot weather.

iii. $\neg(\forall x\ C(x) \rightarrow \neg(T(x) \wedge D(x)),\ x \in \text{People}$
$\equiv \exists x\ \neg(C(x) \rightarrow \neg(T(x) \wedge D(x))),\ x \in \text{People}$
$\equiv \exists x\ \neg(C(x) \rightarrow \neg(T(x) \wedge D(x))),\ x \in \text{People}$
$\equiv \exists x\ (C(x) \wedge \neg(\neg(T(x) \wedge D(x)))),\ x \in \text{People}$
$\equiv \exists x\ (C(x) \wedge (T(x) \wedge D(x))),\ x \in \text{People}$
English: Someone in this class lives in Tucson and has a dog.

3. (12 points)Using the given predicates, complete the specified translations.

$M(x) : x$ is a math course, $x \in$Courses.

$C(x) : x$ is a CS course, $x \in$Courses.

$U(x) : x$ is an upper division course, $x \in$Courses.

$P(x, y) : x$ is a pre-requisite of y, $x, y \in$Courses

(a) (6 points) Translate the following logic statements to English.

i. $\forall x\,(U(x) \wedge C(x)) \rightarrow \exists y\,(P(y, x) \wedge \neg U(y)),\ x, y \in \text{Courses}$
All upper division CS courses have a non-upper division course as a pre-requisite.

ii. $\exists x \exists y\, C(x) \wedge M(y) \wedge P(y, x),\ x, y \in \text{Courses}$
Some CS course has a math course as a pre-requisite.

iii. $\forall x \forall y\,(C(x) \wedge \neg U(x) \wedge C(y) \wedge U(y)) \rightarrow P(x, y),\ x, y \in \text{Courses}$
All upper division CS courses have all non-upper division CS courses as a pre-requisite.

(b) (6 points) Translate the following English statements to logic.

i. All CS courses have a math course as a pre-requisite.
$\forall x\ C(x) \rightarrow \exists y\ M(y) \wedge P(y, x),\ x, y \in \text{courses}$

ii. Some CS courses, that are not in the upper division, have a non-upper division CS course as a pre-requisite.
$\exists x \exists y\ C(x) \wedge \neg U(x) \wedge C(y) \wedge \neg U(y) \wedge P(x,y),\ \ x,y \in$ courses

iii. All CS courses with upper-division pre-requisites are also upper-division courses.
$\forall x \forall y\ (C(x) \wedge U(y) \wedge P(y,x)) \rightarrow U(x),\ \ x,y \in$ courses

4. (2 points) Convert the following quantified predicates to logic statements using only $\wedge, \vee, \neg, \rightarrow$. State the truth value when $P(x) : x < 2,\ x \in \mathbb{Z}$.

   (a) $\forall x\ P(x),\ x \in \{0,1,2\}$
   $P(0) \wedge P(1) \wedge P(2)$, False

   (b) $\exists x\ P(x),\ x \in \{0,1,2\}$
   $P(0) \vee P(1) \vee P(2)$, True

5. (4 points) For each of the following, give the truth value. If true, briefly justify your answer. If false, provide a counterexample. Let $P(x,y) : x * y = 1,\ \ x,y \in \mathbb{R}$

   (a) $\exists x\ P(x,7),\ \ x \in \mathbb{R}$
   True, $x = \frac{1}{7}$

   (b) $\forall x \exists y\ P(x,y),\ \ x,y \in \mathbb{R}$
   False, when $x = 0$, $x * y$ always equals 0.

   (c) $\forall x\ ((x = 0) \vee \exists y\ P(x,y)),\ \ x,y \in \mathbb{R}$
   True. If $x = 0$, the statement is true. For any other x, if $y = \frac{1}{x}$ then $x * y = x * \frac{1}{x} = 1$

   (d) $\exists x \exists y (P(x,x) \wedge P(y,y) \wedge (x \neq y)),\ \ x,y \in \mathbb{R}$
   True. If $x = 1$ and $y = -1$, then $x * x = 1$ and $y * y = 1$.

6. (4 points) Express the following in logic.
   Let $S(x,y) : x$ knows y, $x,y \in$people.

   (a) Everyone knows someone (other than themself).
   $\forall x \exists y\ S(x,y) \wedge (x \neq y),\ \ x,y \in$people

   (b) June only knows 2 people.
   $\exists x \exists y\ S(\text{June},x) \wedge S(\text{June},y) \wedge (x \neq y) \wedge \forall z\ S(\text{June},z) \rightarrow ((z = x) \vee (z = y))$

7. (2 points) Which rule of inference is expressed in the following statements (you only need to provide the name of the rule):

   (a) Jack goes dancing only if it is Friday. Jack does not do homework when it is Friday. Therefore, if Jack goes dancing, he does not do his homework.
   Hypothetical Syllogism.

   (b) All dog breeds are fluffy. Therefore, Chihuahuas are fluffy.
   Universal Instantiation

8. (5 points) For the following premises and conclusion, construct a valid argument that shows uses the hypotheses to reach the conclusion. Be sure to convert all English statements to their corresponding logic statements and justify each step with the rule of inference or logical equivalence used.

(Premise 1) All full time students must be enrolled in at least 12 credits.

(Premise 2) Sarah, a student in this course, is enrolled in fewer than 12 credits.

$\therefore$ Someone in this course is not full time.

Let $F(x)$: x is a full time student. $x \in$ People

$E(x)$: x is enrolled in at least 12 credits. $x \in$ People

$C(x)$: x is in this course. $x \in$ People

(1)		$\forall x\ F(x) \rightarrow E(x),\ x \in \text{People}$	(Given)
(2)		$C(Sarah) \wedge \neg E(Sarah)$	(Given)
(3)		$F(Sarah) \rightarrow E(Sarah)$	(Universal Instantiation of (1))
(4)		$\neg E(Sarah)$	(Simplification (2))
(5)		$\neg F(Sarah)$	(Modus Tollens of (4) and (3))
(6)		$C(Sarah)$	(Simplification of (2))
(7)		$C(Sarah) \wedge \neg F(Sarah)$	(Conjunction of (5) and (6))
(8)	$\therefore$	$\exists x\ C(x) \wedge \neg F(x),\ x \in \text{People}$	(Existential Generalization of (7))

Alternative:

Let $F(x)$: x is full time. $x \in$ People

$S(x)$: x is a student. $x \in$ People

$E(x)$: x is enrolled in at least 12 credits. $x \in$ People

$C(x)$: x is in this course. $x \in$ People

(1)		$\forall x\ (F(x) \wedge S(x)) \rightarrow E(x),\ x \in \text{People}$	(Given)
(2)		$S(Sarah) \wedge C(Sarah) \wedge \neg E(Sarah)$	(Given)
(3)		$(F(Sarah) \wedge S(Sarah)) \rightarrow E(Sarah)$	(Universal Instantiation of (1))
(4)		$\neg E(Sarah)$	(Simplification (2))
(5)		$\neg(F(Sarah) \wedge S(Sarah))$	(Modus Tollens of (4) and (3))
(6)		$\neg F(Sarah) \vee \neg S(Sarah)$	(DeMorgan's Law of (5)
(7)		$S(Sarah)$	(Simplification of (2))
(8)		$\neg F(Sarah)$	(Disjunctive Syllogism of (6) and (7))
(9)		$C(Sarah)$	(Simplification of (2))
(10)		$C(Sarah) \wedge \neg F(Sarah)$	(Conjunction of (8) and (9))
(11)	$\therefore$	$\exists x\ C(x) \wedge \neg F(x),\ x \in \text{People}$	(Existential Generalization of (10))

9. (5 points) Use rules of inference to show that the conclusion follows from the given premises. Be sure to convert all English statements to their corresponding logic statements and justify each step with the rule of inference or logical equivalence used.

(Premise 1) At least 2 (different) people are computer scientists.

(Premise 2) Anyone who doesn't own a computer is not a computer scientist.

(Premise 3) All computer scientists are know each other.

$\therefore$ There are 2 (different) people who own computers and know each other.

Let $C(x)$: x is a computer scientist. $x \in$ People

$O(x)$: x owns a computer. $x \in$ People

$K(x, y)$: x knows y. $x, y \in$ People

(1)		$\exists x \exists y\ (C(x) \wedge C(y) \wedge (x \neq y))$, $x \in$ People	(Given)
(2)		$\forall x\ (\neg O(x) \rightarrow \neg C(x))$, $x \in$ People	(Given)
(3)		$\forall x \forall y\ (C(x) \wedge C(y)) \rightarrow K(x, y)$, $x \in$ People	(Given)
(4)		$C(d) \wedge C(e) \wedge (d \neq e)$	(Existential Instantiation of (1))
(5)		$\neg O(d) \rightarrow \neg C(d)$	(Universal Instantiation of (2)
(6)		$C(d)$	(Simplification of (4))
(7)		$O(d)$	(Modus Tollens of (6) and (5))
(8)		$\neg O(e) \rightarrow \neg C(e)$	(Universal Instantiation of (2)
(9)		$C(e)$	(Simplification of (4))
(10)		$O(e)$	(Modus Tollens of (6) and (5))
(11)		$C(d) \wedge C(e)$	(Conjunction of (6) and (9))
(12)		$(C(d) \wedge C(e)) \rightarrow K(d, e)$	(Universal Instantiation of (3))
(13)		$K(d, e)$	(Modus Ponens of (11) and (12))
(14)		$(C(e) \wedge C(d)) \rightarrow K(e, d)$	(Universal Instantiation of (3))
(15)		$K(e, d)$	(Modus Ponens of (11) and (14))
(16)		$(d \neq e)$	(Simplification of (4)
(17)		$O(d) \wedge O(e) \wedge K(d, e) \wedge K(e, d) \wedge (d \neq e)$	(Conjunction of (7),(10),(13),(15),(16)
(18)	$\therefore$	$\exists x \exists y O(x) \wedge O(y) \wedge K(x, y) \wedge K(y, x) \wedge (x \neq y)$, $x \in$ People	(Existential Generalization)

10. (3 points) Is the following argument valid? If yes, give the logical rules used to reach the conclusion. If not, explain why not.

(Premise 1) All Montana residents love skiing

(Premise 2) Jodi is not a Montana resident.

$\therefore$ Jodi hates skiing.

It is not valid. This is an example of Denying the hypothesis. Premise one is true if: Jodi is a Montana resident and lives skiing, Jodi is not a Montana resident and loves skiing, and Jodi is not a Montana resident and does not love skiing. Just because Jodi is not a Montana resident, does not necessarily mean she doesn't like skiing.