CSc 245 Discrete Structures - Summer 2021

Homework #6

Due: July 23rd, 2021 by 11:59 p.m (MST).
(70 points)

Instructions:

1. **Homework assignments are to be completed individually, not in groups.**
2. If you need help, take advantage of Piazza and office hours.
3. Assignments are to be submitted in PDF form via Gradescope. They may be typed (which is preferable and strongly recommended) or handwritten with each page scanned or photographed and compiled into a single PDF.
4. If you choose to handwritten your assignments, please write neatly. Illegible assignments may not be graded.
5. Extra credit will be given for typed homework. To make this easier, a `Latex` template will be provided for each assignment.
6. Show your work (when appropriate) for partial credit!

Part 1: Functions:

1. (4 points) For each of the following relations, determine if it is a function. Justify your answer.
   (a) $f(x) = \frac{1}{x^3}$ from $\mathbb{Z}$ to $\mathbb{R}$
   (b) $f(x) = \sqrt{x}$ from $\mathbb{Z}^+$ to $\mathbb{R}$
2. (4 points) For each of the following functions, give the domain, codomain, and range.
   (a) $f(x) = x\%5$.
   (b) The function that, given a bit string, returns the length of the bit string.
3. (4 points) Evaluate each of the following:
   (a) $\lceil -3.5 \rceil$
   (b) $\lceil -2.2 * \lceil -\frac{5}{2} \rceil \rceil$
   (c) $\lfloor 2.99 \rfloor$
   (d) $\lfloor \frac{4}{3} + \lceil \frac{1}{3} \rceil \rfloor$
4. (6 points) For each of the following functions determine if it is injective. Justify your answer.
   (a) $f(n) = n^5$ from $\mathbb{Z}$ to $\mathbb{Z}$
   (b) $f(n) = |n| + 1$ from $\mathbb{Z}$ to $\mathbb{Z}^+$
   (c) $f(n) = n + 3$ from $\mathbb{Z}$ to $\mathbb{Z}$

5. (6 points) Determine if each of the following functions from question 4 are surjective. Justify your answer.
   (a) $f(n) = n^5$ from $\mathbb{Z}$ to $\mathbb{Z}$
   (b) $f(n) = |n| + 1$ from $\mathbb{Z}$ to $\mathbb{Z}^+$
   (c) $f(n) = n + 3$ from from $\mathbb{Z}$ to $\mathbb{Z}$
6. (6 points) Which of the functions from questions 4 and 5 are bijective? Briefly justify your answer.
   (a) $f(n) = n^5$ from $\mathbb{Z}$ to $\mathbb{Z}$
   (b) $f(n) = |n| + 1$ from $\mathbb{Z}$ to $\mathbb{Z}^+$
   (c) $f(n) = n + 3$ from from $\mathbb{Z}$ to $\mathbb{Z}$
7. (4 points) For each of the following, give an example of a function from $\mathbb{Z}^*$ to $\mathbb{Z}^*$ that satisfies the specified properties. You may not use the functions from the previous three problems.
   (a) One-to-one but not onto
   (b) Onto but not one-to-one
8. (4 points) Let C be the set of CS Faculty. Consider the following functions whose domains are C. Answer the following for each function: (i) Under what conditions is the function one-to-one? (ii) Under what conditions is the function onto?
   (a) The function f from C to O that assigns faculty to offices, where O is the set of offices on the 7th floor of Gould Simpson.
   (b) The function g from C to S that assigns faculty to the courses they will teach this semester, where S is the set of courses offered by the CS department. Assume each faculty teaches exactly one course (otherwise it's not a function!)
9. (4 points) Prove or disprove the following conjecture: $\lceil 3x \rceil = 3 \cdot \lceil x \rceil$.
10. (8 points) Prove the following conjecture: $\lfloor x \rfloor + \lfloor y \rfloor \leq \lfloor x + y \rfloor$. (Hint: Use the fact that, for any real number x, $x = m + d$ where $m \in \mathbb{Z}$, $d \in \mathbb{R}$ with $m \leq x < m + 1$ and $0 \leq d < 1$).

Part 2: Integers:

12. (4 points) For each of the following divisions, give the quotient and remainder.
    (a) 85 divided by 12
    (b) 230 divided by 15
13. (4 points) Give the prime factorization of the following integers.
    (a) 168
    (b) 90
14. (4 points) Determine whether each of theses sets are pairwise relatively prime. Show your work.
    (a) 60, 91, 11
    (b) 105, 52, 13

15. (4 points) Give the GCD and LCM for each of the following pairs of integers.

    (a) $2^2 \cdot 3^2 \cdot 5 \cdot 13$, $2 \cdot 5^2 \cdot 7 \cdot 11$

    (b) $3^2 \cdot 7 \cdot 11$, $3 \cdot 5 \cdot 11^2$

16. (4 points) Given the following values of ab and either the GCD or the LCM, compute the LCM (if you were given the GCD) or the GCD (if you were given the LCM).

    (a) $ab = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2 \cdot 11$, GCD($a$,$b$) $= 2 * 3 * 7$

    (b) $ab = 2^2 \cdot 13^2 \cdot 17$, LCM($a$,$b$) $= 2 * 13^2 * 17$