CSc 245 Discrete Structures - Summer 2021

Quiz #2

Due: June 22nd, 2021 by 11:59 pm (MST)

Solutions

1. **Truth of Quantified Predicates**(2pts): For each of the following, determine if it is true or false. Briefly justify your answer. Let $P(x, y) : x\%y = 0,\ x, y \in \mathbb{Z}$

   (a) $\exists x \forall y\ P(x, y),\ x, y \in \mathbb{Z}$

   False. When $y = 0$, $x\%y$ is never defined.

   (b) $\forall x \exists y\ P(x, y),\ x, y \in \mathbb{Z}$

   True. If y is 1, x, or any multiple of x, then $x\%y = 0$.

2. **Translating Quantifications**(4pts): Consider the following predicate:

   - $T(x, y) : x$ has traveled to y, $x \in$ People,$y \in$ Country
   - $C(a) : a$ is in this course, $a \in$People
   - $E(b) : b$ is in Europe, $b \in$Country

   (a) Convert the following statements into logic:

   i. Someone in this course has traveled to every country in Europe.

   $\exists x \forall y\ C(x) \wedge (E(y) \rightarrow T(x, y))\ \ x \in$ People, $y \in$ Countries

   ii. Nathan, a student in this course, has traveled to Mexico but not to Canada.

   $C(Nathan) \wedge (P(Nathan, Mexico) \wedge \neg T(Nathan, Canada)$

   (b) Convert the following statements into conversational English:

   i. $\exists x \exists y\ T(\text{Sandy}, x) \wedge T(\text{Sandy}, y) \wedge (x \neq y),\ \ x, y \in$ Countries

   Sandy has traveled to at least 2 countries.

   ii. $\forall x E(x) \rightarrow \exists y(C(y) \wedge T(y, x)), x \in$ Countries, $y \in$ People

   Every country in Europe has been visited by at least one person in this course.

3. **Arguments** (4pts): Consider the following premises and conclusion. First identify the atomic propositions, assign them labels and convert the following statements into logic. Then use the Rules of Inference to prove that the conclusion must follow from the premises. Label the rule used at each step! Note, this question does not require any predicates, only atomic propositions.

Jacob programs in Python and Java.
If Jacob programs in Python, then he doesn't program in Java or he isn't programming for homework.
If Jacob is in a CS course, then he is programming for homework.

$\therefore$ Jacob is not in a CS course.

p : Jacob programs in Python.
j : Jacob programs in Java.
c : Jacob is in a CS course
h : Jacob is programming for homework

(1)	$p \wedge j$	(Given)
(2)	$p \rightarrow \neg j \vee \neg h$	(Given)
(3)	$c \rightarrow h$	(Given)
(4)	p	(Simplification of (1))
(5)	$\neg j \vee \neg h$	(Modus Ponens of (2) and (4))
(6)	j	(Simplification of (1))
(7)	$\neg h$	(Disjunctive Syllogism of (6) and (5))
(8)	$\therefore \neg c$	(Modus Tollens (3) and (7))