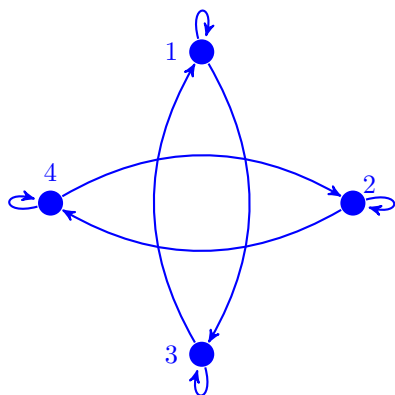# Quiz #4

Due: July 13th, 2021 by 11:59 pm (MST)

1. Let $R = \{(x, y) \mid x - y \text{ is even. } x, y \in H\}$ on $H = \{1, 2, 3, 4\}$

   (a) Draw the digraph of $R$.

   

   (b) For each of the following, specify true or false. If true, briefly explain. If false, specify the pairs that need to be added/removed to satisfy the stated property.

   - $R$ is reflexive.
     True. $x - x = 0$ which is even.

   - $R$ is irreflexive.
     False. We need to remove $(1, 1)$, $(2, 2)$, $(3, 3)$ and $(4, 4)$.

   - $R$ is symmetric.
     True. If $x - y$ is even then for some integer $k$, $x - y = 2k$. $y - x = -(x - y) = -2k = 2(-k)$ which is also even.

   - $R$ is anitsymmetric.
     False. We need to remove one of $(4, 2)$ and $(2, 4)$, and one of $(3, 1)$ and $(1, 3)$.

   - $R$ is transitive.
     True. $(1, 3)$ and $(3, 1) \Rightarrow (1, 1)$ which is in the relation.
     $(3, 1)$ and $(1, 3) \Rightarrow (3, 3)$ which is in the relation.
     $(2, 4)$ and $(4, 2) \Rightarrow (2, 2)$ which is in the relation.
     $(4, 2)$ and $(2, 4) \Rightarrow (4, 4)$ which is in the relation.

   (c) What type of relation is $R$?

   A. Equivalence Relation
   B. Weak Partial Ordering
   C. Strong Partial Ordering
   D. Total Ordering
   E. None of the above.