CSc 245 - Introduction to Discrete Structures
Summer 2020

Name: ____

Final

Directions

1. Write your name at the top
2. This exam is open book, closed internet. Looking for answers online or asking for/using answers written by other people is a violation of academic integrity.
3. Exams must be completed **individually**. You may not discuss questions with other students or on Piazza.
4. Clarifications may be asked on Piazza, but please make them private. We will decide if they should be made public.
5. Response may be typed or handwritten. Make sure your responses are legible!
6. Make your answers as precise and concise and to the point as possible, while still answering the questions asked.
7. Show your work, where appropriate, for potential partial credit. Vague, incomplete, and/or ambiguous answers will not receive full credit.

1. (5 Points) For each of the following, specify whether it is a proposition.

   (a) $5x > 0, x \in \mathbb{Z}$

   (b) Don't run with scissors.

   (c) Mushrooms are tasty.

   (d) 5+5 =2

   (e) This sentence is a lie.

2. (2 points) List the first four terms of the sequence $s_n = \lceil \frac{n}{2} \rceil + \frac{3n}{4}$ where $n \geq 5$.

3. (4 points) Find the GCD and LCM of 54 and 72

4. (5 points) Write the truth table for $\neg(\neg p \vee q) \wedge \neg(p \rightarrow q)$. Be sure to include all columns.

5. (4 points) Let $P(x, y) = x|y$. Express the truth value for the following two quantification's. Briefly explain your answer.

   (a) $\forall y \exists x P(x, y)$, $x, y \in \mathbb{Z}$

   (b) $\exists y \forall x P(x, y)$, $x, y \in \mathbb{Z}$

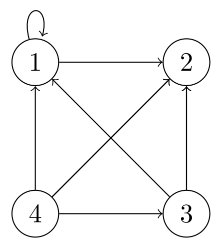
6. (6 points)Use rules of inference to show that conclusion follows from the given premises. First identify and label all predicates. Then convert the premises and conclusion to their corresponding logic statements using those predicates. Then apply rules of inference to the premises to reach the given conclusion. Note: The given premises alone do not give the conclusion, you must apply rules of inference to them to reach the conclusion.

   All dogs like treats.

   Rufus (a dog) likes to chew bones.

   $\therefore$ There is a dog who likes treats and chewing bones.

7. (12 points) Given the graph below of the relation R, answer the following:



(a) Give the matrix representation of the relation.

(b) Determine if the relation is a weak partial ordering, strong partial ordering and/or total ordering. Justify your answer.

(c) How can we change the graph to represent the inverse of the relation?

(d) Is the relation a function? Explain why or why not.

8. (8 points) For each of the following functions from $\mathbb{Z} \to \mathbb{Z}$, determine if they are injective, surjective, and/or bijective. Justify your answers.

   (a) $f(x) = x^2 + 1$

   (b) $f(x) = x^3 + 8$

   (c) $f(x) = x - 2$

   (d) $f(x) = \lfloor \frac{x}{3} \rfloor$

9. (10 points) Use a proof by contradiction to show that if $n\%5 \neq 0$ then n is not the sum of five consecutive integers.

10. (6 points) Determine if each of the following sets are countable or uncountable. Justify your answer. For those that are countable, create a bijective mapping from the set to either the positive integers or non-negative integers.

    (a) The negative integers

    (b) The even integers

    (c) The real numbers between 0 and $\frac{2}{3}$

11. (9 points) The animal shelter has 30 dogs. 6 of these dogs are part or whole Golden Retriever, 11 are part or whole Husky, and 8 are part or whole Great Dane. 2 of the dogs are both Golden Retriever and Husky and 3 of the dogs are both Husky and Great Dane. Only 1 of the dogs was a mix of Great Dane and Golden Retriever. None of the dogs are a mix of all three breeds.

   (a) How many dogs are whole Husky?

   (b) What is the cardinality of the union of the sets of dogs that are part or whole Great Dane and part or whole Husky?

   (c) How many dogs are neither Golden Retriever, Husky nor Great Dane?

12. (10 points) A grocery store sells flour in 3lb and 8lb bags. Use induction to prove that a customer could buy any amount (in pounds) of flour greater than 13 lbs.

13. (18 points)Rodger's Board & Brew is a brewery that also has a collection of board games for patrons to play while visiting. They have 5 different categories of board games: Party, Strategy, Dice, Card, and Bluffing, and 6 types of beer: IPA, Amber, Sour, Wheat, Pale Ale, and Cream Ale. Patrons can either buy a beer or pay $5 to play up to 3 games. After 3 games, they must buy another beer or pay another $5 to play up to three more, and so on.

    (a) David buys 2 beers to play 6 games. In how many ways can David select the 2 beers and 6 games?

    (b) David likes strategy games but can play no more than 1 in a night. How many ways can he select 7 games given that he selects no more than 1 strategy game?

    (c) David is not a fan of bluffing games or sour beers. If he buys 3 beers to play 8 games, how many ways can he choose those 3 beers and 8 games if he excludes bluffing games and sour beers?

    (d) Lisa buys a 2 ounce pour of each beer so she can try all of them. How many ways can she arrange her drinks?

    (e) After tasting all of the beer she decides to get cans of 3 of them to take home with her. How many ways are there for her to select and arrange her three cans?

    (f) A large party arrives and is trying to decide what kind of game to play. Each category is voted for by at least 2 people. At least how many people are in the group?

14. (18 points) Algorithms

(a) Given the recursive algorithm below give the recursive definition.

```
subprogram triplesum (given: n)
        returns: sum from 1 through n of 3i

    if n is 1, return 3
    otherwise
        answer <-- triplesum(n-1)+3*n
        return answer
    end if

end subprogram
```

(b) Prove that the algorithm above returns $\sum_{i=1}^{n} 3n$.

15. (8 points)Recurrence Relations

   (a) Find the solution to the linear homogeneous recurrence relation with constant coefficient of degree 2: $R(n) = 4R(n-2)$ where $R(0) = 2$ and $R(1) = 4$