CSc 245 - Introduction to Discrete Structures
Summer 2020

Name: Your name here

Midterm

Released July 9th, Due July 10th by 5pm

Directions

1. Write your name at the top
2. This exam is open book, closed internet. Looking for answers online or asking for/using answers written by other people is a violation of academic integrity.
3. Exams must be completed **individually**. You may not discuss questions with other students or on Piazza.
4. Clarifications may be asked on Piazza, but please make them private. We will decide if they should be made public.
5. Response may be typed or handwritten. Make sure your responses are legible!
6. Make your answers as precise and concise and to the point as possible, while still answering the questions asked.
7. Show your work, where appropriate, for potential partial cblueit. Vague, incomplete, and/or ambiguous answers will not receive full cblueit.
8. Submit your responses on Gradescope by Friday (7/10) at 5pm MST (Tucson time).

1. (6 points) Math Review

   (a) Give 3 *different* integers that are congruent to 23 modulo 5.
   Example answer: 3, 8, 13

   (b) Evaluate $36|6$ and $54|9$. Explain your answers.
   Both are False, 36 does not divide 6 and 54 does not divide 9.

   (c) Convert 110101 to Decimal. Show your work.
   $2^0 * 1 + 2^1 * 0 + 2^2 * 1 + 2^3 * 0 + 2^4 * 1 + 2^5 * 1$
   $= 1 + 4 + 16 + 32$
   $= 53$

2. (6 points) Complete parts (a) and (b) using the given propositions below.
   b : my bread is under-baked. p : my bread is under-proved.
   t : I start baking before noon g : the bread is good.

   (a) Convert to Logic: If my bread is under-baked or under-proved, then the bread is not good.

   $(b \vee p) \rightarrow \neg g$

   (b) Convert to English: $\neg t \rightarrow ((b \vee p) \wedge \neg g)$

   If I don't start baking before noon, then my bread will be under-baked or under-proved and won't be good.

3. (3 points) For each of these statements, write the antecedent and consequent in the spaces.

   (a) I will bake bread unless I run out of time.

   Antecedent: <u>If I don't run out of time</u> Consequent: <u>I will bake bread</u>

   (b) James will ride his bike only if it is warm outside.

   Antecedent: <u>James will ride his bike</u> Consequent: <u>it is warm outside</u>

   (c) I will make pizza provided that I have all of the ingredients.

   Antecedent: <u>If I have all of the ingredients</u> Consequent: <u>I will make pizza</u>

4. (4 points) Given $P(x, y) : x\%y = 0$ state the truth values of the following. Briefly justify your answer.

   (a) $\exists x \forall y \, P(x, y)$, $x, y \in \mathbb{Z}$.

   False, the only potential value of x that would where $x\%y = 0$ would be $x = 0$ but when $y = 0$, $x\%y$ is not defined (can't divide by zero)

   (b) $\forall x \exists y \, P(x, y)$, $x, y \in \mathbb{Z}$.

   True, when $x = y$, $x\%y = 0$.

5. (3 points) What rule of inference is used in each of the following arguments? Note: these statements each correspond to a *single* rule, you simply need to state the name of the rule.

   (a) French bread contains yeast. Therefore some breads contain yeast.

   Existential Generalization

   (b) All harry potter books are good. Therefore Harry potter and the Prisoner of Azkaban is good.

   Universal instantiation

   (c) I wash the dishes or read my book. I don't wash the dishes. Therefore, I read my book.

   Disjunctive Syllogism

6. (4 points) Give a counterexample to disprove the conjecture: given n is even and k is an integer, if nk is even, then k must be even.

   Example answer: $n = 4$ which is even and $k = 3$ is odd but $nk = 12$ which is even.

7. (2 points) Given $A = \begin{bmatrix} 1 & 2 \\ 4 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 0 \\ 5 & 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 7 & 1 \end{bmatrix}$

   (a) What pairs of sets can be multiplied together? (note: order matters!)

   $A \cdot B$, $B \cdot A$, $B \cdot C$, $C \cdot B$

   (b) What pairs can be summed together?

   $A + C$

8. (4 points) Given $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, evaluate the following:

(a) $A \wedge B$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b) $A \vee B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(c) $A \odot B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(d) $A \cdot B$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

9. (4 points) Let A be the set of all positive integers less than 6. Let $B = \{4, 5, 7, 8\}$

(a) Express A in set builder notation.

$\{x | 0 < x < 6,\ x \in \mathbb{Z}\}$

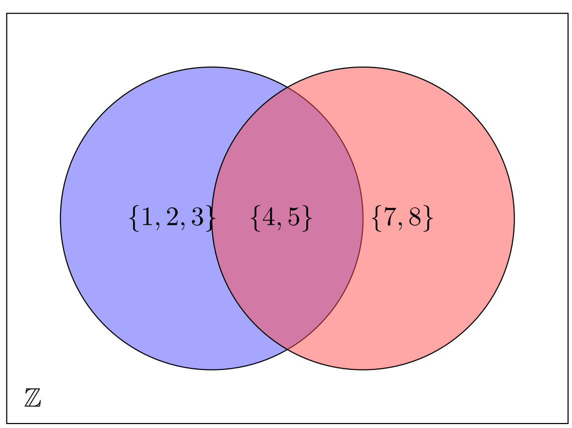
(b) What is $|A|$?

$|A| = 5$

(c) What is $A \cap B$

$A \cap B = \{4, 5\}$

(d) Draw the Venn Diagram of A and B.



10. (15 points) Consider the following compound proposition: $\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$

(a) Show that $\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$ is a tautology using truth tables.

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg q \wedge (p \vee q)$	$q \wedge \neg p$	$\neg(q \wedge \neg p)$	$\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$
T	T	F	F	T	F	F	T	T
T	F	F	T	T	T	F	T	T
F	T	T	F	T	F	T	F	T
F	F	T	T	F	F	F	T	T

(b) Now show that $\neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p)$ is a tautology using equivalences.

$$
\begin{array}{lll}
 & \neg q \wedge (p \vee q) \rightarrow \neg(q \wedge \neg p) & \\
\equiv & ((\neg q \wedge p) \vee (\neg q \wedge q)) \rightarrow \neg(q \wedge \neg p) & \text{(Distributive Law)} \\
\equiv & (\neg q \wedge p) \vee F \rightarrow \neg(q \wedge \neg p) & \text{(Negation Law)} \\
\equiv & (\neg q \wedge p) \rightarrow \neg(q \wedge \neg p) & \text{(Identity Law)} \\
\equiv & \neg(\neg q \wedge p) \vee \neg(q \wedge \neg p) & \text{(Law of Implication)} \\
\equiv & (q \vee \neg p) \vee \neg(q \wedge \neg p) & \text{(De Morgan)} \\
\equiv & q \vee \neg p \vee \neg q \vee p & \text{(De Morgan)} \\
\equiv & q \vee \neg q \vee \neg p \vee p & \text{(Commutative Law)} \\
\equiv & T \vee \neg p \vee p & \text{(Negation Law)} \\
\equiv & T & \text{(Domination Law)}
\end{array}
$$

11. (15 points) Use the following predicates to complete parts (a)-(c).

$G(x)$: x has a garden $P(x, y)$: x has y planted in their garden
$V(y)$: y is a vegetable $H(y)$: y is an herb.

(a) Convert to Logic

i. Everyone with a garden has a vegetable in their garden.

$\forall x\ G(x) \rightarrow \exists y\ (V(y) \wedge P(x, y))$, $x \in$ People, $y \in$ Plants

ii. Ian has a garden with the herb Rosemary planted in it.

$H(\text{Rosemary}) \wedge G(\text{Ian}) \wedge P(\text{Ian, Rosemary})$

(b) Convert to English: $H(\text{Thyme}) \wedge \forall x(G(x) \rightarrow P(x,\text{Thyme}))$, $x \in$ People .

Everyone with a garden has the herb Thyme in it.

(c) Express the negation of the statement in part (b) in:

i. Logic

$\neg H(\text{Thyme}) \vee \exists x\ G(x) \wedge \neg P(x,\text{Thyme})$, $x \in$ People

ii. English

Someone does not have the herb Thyme in their garden.

12. (10 points) Use rules of inference to show that conclusion follows from the given premises. First identify and label all predicates. Then convert the premises and conclusion to their corresponding logic statements using those predicates. Then apply rules of inference to the premises to reach the given conclusion. Note: The given premises alone do not give the conclusion, you must apply rules of inference to them to reach the conclusion.

Everyone who drives a car has a license.

Tina is a UA student and drives a car.

$\therefore$ A UA student has a license.

$D(x) : x$ drives a car

$L(x) : x$ has a license

$U(x) : x$ is a UA student

Translating our givens and conclusion:

$\forall x\, D(X) \to L(x), x \in \text{People}$

$D(\text{Tina}) \land U(\text{Tina})$

$\exists x\, U(x) \land L(x)$

(1)	$\forall x\, D(X) \to L(x), x \in \text{People}$	(Given)
(2)	$D(\text{Tina}) \land U(\text{Tina})$	(Given)
(3)	$D(\text{Tina}) \to L(\text{Tina})$	(Universal Instantiation of (1))
(4)	$D(\text{Tina})$	(Simplification of (2))
(5)	$L(\text{Tina})$	(Modus Ponens of (3) and (4))
(6)	$U(\text{Tina})$	(Simplification of (2))
(7)	$U(\text{Tina}) \land L(\text{Tina})$	(Conjunction of (5) and (6))
(8)	$\exists x\, U(x) \land L(x)$	(Existential Generalization of (7))

13. (14 points) Given that n be an odd integer and k be an integer. Prove that an integer k is even **if and only if** kn is even

Proof: Because this is a biconditional, we need to prove it in both directions. We will do the "only if" direction directly, and the "if" direction through contraposition.

$\rightarrow$ (Direct) We want to show that given that n is odd, if k is even, then kn is even.

Assume k is even. We know that n is odd.

Therefore there exists integers j and l such that $k = 2j$ and $n = 2l + 1$.

Substituting we get, $nk = (2j)(2l + 1) = 4jl + 2j = 2(2jl + j)$

If we let $m = 2jl + j$, nk then has the form $2m$. m will be an integer since all of its operands are integers and the product or sum of integers is an integer. Therefore, nk has the form of an even number $(2m)$ and thus, is even.

Therefore if k is even, then nk is even.

$\leftarrow$ (Contraposition) In this direction, we need to prove that given that n is odd, if nk is even, then k is even. We will do this through contraposition. So we will instead show that given that n is odd, if k is odd, then nk is odd.

We know that n is odd. Assume that k is also odd.

By definition, there exist integers j, l such that $k = 2j + 1$ and $n = 2l + 1$.

Substituting we get, $nk = (2j + 1)(2l + 1) = 4jl + 2j + 2l + 1 = 2(2jl + j + l) + 1$

If we let $m = 2jl + j + l$, nk then has the form $2m + 1$. m will be an integer since all of its operands are integers and the product or sum of integers is an integer. Therefore, nk has the form of an odd number $(2m + 1)$ and thus, is odd.

Thus we have proved the contrapositive.

Therefore if nk is even, then k is even.

We have now shown both directions of the conjecture. Therefore given that n is an odd integer and k is an integer, k is even **if and only if** kn is even

14. (10 points) Let A,B, and C be sets. Prove that if $A \cup B = B \cup C$ and $A \cap C = B \cap C$, then $A \subseteq B$ (Hint: it may be easier to not use a direct proof!)

Proof (Contradiction): Assume $A \cup B = B \cup C$ and $A \cap C = B \cap C$, but $A \not\subseteq B$.

By definition of a subset, if $A \not\subseteq B$, then there exists an $x \in A$ such that $x \notin B$.

Since $x \in A$, $x \in A \cup B$ (by definition of a union).

Because $A \cup B = B \cup C$, x must be in $B \cup C$. This means that $x \in B$ or $x \in C$. But we know $x \notin B$. Therefore, x must be in C.

Since $x \in A$ and $x \in C$, $x \in A \cap C$.

We know that $A \cap C = B \cap C$, so since $x \in A \cap C$, it must also be in $B \cap C$.

However, by definition of an intersection, this means that $x \in B$ and $x \in C$. But we assumed that $x \notin B$. This gives us a contradiction.

Therefore, if $A \cup B = B \cup C$ and $A \cap C = B \cap C$, then $A \subseteq B$